ANTON BIVENS DAVIS

EARLY TRANSCENDENTALS 10TH EDITION

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10th EDITION CALCULUS

EARLY TRANSCENDENTALS

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**About HOWARD ANTON** Howard Anton obtained his B.A. from Lehigh University, his M.A. from the University of Illinois,

and his Ph.D. from the Polytechnic University of Brooklyn, all in mathematics. In the early 1960s he worked for Burroughs Corporation and Avco Corporation at Cape Canaveral, Florida, where he was involved with the manned space program. In 1968 he joined the Mathematics Department at Drexel University, where he taught full time until 1983. Since that time he has been an Emeritus Professor at Drexel and has devoted the majority of his time to textbook writing and activities for mathematical associations. Dr. Anton was president of the EPADEL section of the Mathematical Association of America (MAA), served on the Board of Governors of that organization, and guided the creation of the student chapters of the MAA. He has published numerous research papers in functional analysis, approximation theory, and topology, as well as pedagogical papers. He is best known for his textbooks in mathematics, which are among the most widely used in the world. There are currently more than one hundred versions of his books, including translations into Spanish, Arabic, Portuguese, Italian, Indonesian, French, Japanese, Chinese, Hebrew, and German. His textbook in linear algebra has won both the Textbook Excellence Award and the McGuffey Award from the Textbook Author’s Association. For relaxation, Dr. Anton enjoys traveling and photography.

**About IRL BIVENS** Irl C. Bivens, recipient of the George Polya Award and the Merten M. Hasse Prize for Expository

Writing in Mathematics, received his A.B. from Pfeiffer College and his Ph.D. from the University of North Carolina at Chapel Hill, both in mathematics. Since 1982, he has taught at Davidson College, where he currently holds the position of professor of mathematics. A typical academic year sees him teaching courses in calculus, topology, and geometry. Dr. Bivens also enjoys mathematical history, and his annual History of Mathematics seminar is a perennial favorite with Davidson mathematics majors. He has published numerous articles on undergraduate mathematics, as well as research papers in his specialty, differential geometry. He has served on the editorial boards of the MAA Problem Book series, the MAA Dolciani Mathematical Expositions series and *The College Mathematics Journal*. When he is not pursuing mathematics, Professor Bivens enjoys reading, juggling, swimming, and walking.

**About STEPHEN DAVIS** Stephen L. Davis received his B.A. from Lindenwood College and his Ph.D. from Rutgers University in mathematics. Having previously taught at Rutgers University and Ohio State University, Dr. Davis came to Davidson College in 1981, where he is currently a professor of mathematics. He regularly teaches calculus, linear algebra, abstract algebra, and computer science. A sabbatical in 1995–1996 took him to Swarthmore College as a visiting associate professor. Professor Davis has published numerous articles on calculus reform and testing, as well as research papers on finite group theory, his specialty. Professor Davis has held several offices in the Southeastern section of the MAA, including chair and secretary-treasurer and has served on the MAA Board of Governors. He is currently a faculty consultant for the Educational Testing Service for the grading of the Advanced Placement Calculus Exam, webmaster for the North Carolina Association of Advanced Placement Mathematics Teachers, and is actively involved in nurturing mathematically talented high school students through leadership in the Charlotte Mathematics Club. For relaxation, he plays basketball, juggles, and travels. Professor Davis and his wife Elisabeth have three children, Laura, Anne, and James, all former calculus students.

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***To*my wife Pat and my children: Brian, David, and Lauren**

***In Memory of* my mother Shirley my father Benjamin my thesis advisor and inspiration, George Bachman my benefactor in my time of need, Stephen Girard (1750–1831) —*HA***

***To*my son Robert —*IB***

***To*my wife Elisabeth my children: Laura, Anne, and James —*SD***

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PREFACE

This tenth edition of *Calculus* maintains those aspects of previous editions that have led to the series’ success—we continue to strive for student comprehension without sacrificing mathematical accuracy, and the exercise sets are carefully constructed to avoid unhappy surprises that can derail a calculus class.

All of the changes to the tenth edition were carefully reviewed by outstanding teachers comprised of both users and nonusers of the previous edition. The charge of this committee was to ensure that all changes did not alter those aspects of the text that attracted users of the ninth edition and at the same time provide freshness to the new edition that would attract new users.

**NEW TO THIS EDITION**

• Exercise sets have been modified to correspond more closely to questions in*WileyPLUS*. In addition, more *WileyPLUS* questions now correspond to specific exercises in the text.

• New applied exercises have been added to the book and existing applied exercises have been updated.

• Where appropriate, additional skill*/*practice exercises were added.

**OTHER FEATURES**

**Flexibility** This edition has a built-in flexibility that is designed to serve a broad spectrum of calculus philosophies—from traditional to “reform.” Technology can be emphasized or not, and the order of many topics can be permuted freely to accommodate each instructor’s specific needs.

**Rigor** The challenge of writing a good calculus book is to strike the right balance between rigor and clarity. Our goal is to present precise mathematics to the fullest extent possible in an introductory treatment. Where clarity and rigor conflict, we choose clarity; however, we believe it to be important that the student understand the difference between a careful proof and an informal argument, so we have informed the reader when the arguments being presented are informal or motivational. Theory involving *ε*-*δ* arguments appears in separate sections so that they can be covered or not, as preferred by the instructor.

**Rule of Four** The “rule of four” refers to presenting concepts from the verbal, algebraic, visual, and numerical points of view. In keeping with current pedagogical philosophy, we used this approach whenever appropriate.

**Visualization** This edition makes extensive use of modern computer graphics to clarify concepts and to develop the student’s ability to visualize mathematical objects, particularly those in 3-space. For those students who are working with graphing technology, there are**vii**

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**viii Preface**

many exercises that are designed to develop the student’s ability to generate and analyze mathematical curves and surfaces.

**Quick Check Exercises** Each exercise set begins with approximately five exercises (answers included) that are designed to provide students with an immediate assessment of whether they have mastered key ideas from the section. They require a minimum of computation and are answered by filling in the blanks.

**Focus on Concepts Exercises** Each exercise set contains a clearly identified group of problems that focus on the main ideas of the section.

**Technology Exercises** Most sections include exercises that are designed to be solved using either a graphing calculator or a computer algebra system such as *Mathematica*, *Maple*, or the open source program *Sage*. These exercises are marked with an icon for easy identification.

**Applicability of Calculus** One of the primary goals of this text is to link calculus to the real world and the student’s own experience. This theme is carried through in the examples and exercises.

**Career Preparation** This text is written at a mathematical level that will prepare stu- dents for a wide variety of careers that require a sound mathematics background, including engineering, the various sciences, and business.

**Trigonometry Review** Deficiencies in trigonometry plague many students, so we have included a substantial trigonometry review in Appendix B.

**Appendix on Polynomial Equations** Because many calculus students are weak in solving polynomial equations, we have included an appendix (Appendix C) that reviews the Factor Theorem, the Remainder Theorem, and procedures for finding rational roots.

**Principles of Integral Evaluation** The traditional Techniques of Integration is entitled “Principles of Integral Evaluation” to reflect its more modern approach to the material. The chapter emphasizes general methods and the role of technology rather than specific tricks for evaluating complicated or obscure integrals.

**Historical Notes** The biographies and historical notes have been a hallmark of this text from its first edition and have been maintained. All of the biographical materials have been distilled from standard sources with the goal of capturing and bringing to life for the student the personalities of history’s greatest mathematicians.

**Margin Notes and Warnings** These appear in the margins throughout the text to clarify or expand on the text exposition or to alert the reader to some pitfall.

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SUPPLEMENTS

The **Student Solutions Manual**, which is printed in two volumes, provides detailed solu- tions to the odd-numbered exercises in the text. The structure of the step-by-step solutions matches those of the worked examples in the textbook. The Student Solutions Manual is also provided in digital format to students in *WileyPLUS*.

Volume I (Single-Variable Calculus, Early Transcendentals) ISBN: 978-1-118-17381-7 Volume II (Multivariable Calculus, Early Transcendentals) ISBN: 978-1-118-17383-1

**The Student Study Guide** is available for download from the book companion Web site at www.wiley.com*/*college*/*anton or at www.howardanton.com and to users of *WileyPLUS*.

The **Instructor’s Solutions Manual**, which is printed in two volumes, contains detailed solutions to all of the exercises in the text. The Instructor’s Solutions Manual is also available in PDF format on the password-protected Instructor Companion Site at www.wiley.com*/* college*/*anton or at www.howardanton.com and in *WileyPLUS*.

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The **Instructor’s Manual** suggests time allocations and teaching plans for each section in the text. Most of the teaching plans contain a bulleted list of key points to emphasize. The discussion of each section concludes with a sample homework assignment. The Instructor’s Manual is available in PDF format on the password-protected Instructor Companion Site at www.wiley.com*/*college*/*anton or at www.howardanton.com and in *WileyPLUS*.

The **Web Projects (Expanding the Calculus Horizon)** referenced in the text can also be downloaded from the companion Web sites and from *WileyPLUS*.

Instructors can also access the following materials from the book companion site or *WileyPLUS*:

• **Interactive Illustrations** can be used in the classroom or computer lab to present and explore key ideas graphically and dynamically. They are especially useful for display of three-dimensional graphs in multivariable calculus.

• The **Computerized Test Bank** features more than 4000 questions—mostly algorithmi- cally generated—that allow for varied questions and numerical inputs.

• The **Printable Test Bank** features questions and answers for every section of the text.

• **PowerPoint lecture slides** cover the major concepts and themes of each section of the book. Personal-Response System questions (“Clicker Questions”) appear at the end of each PowerPoint presentation and provide an easy way to gauge classroom understanding.

• **Additional calculus content** covers analytic geometry in calculus, mathematical mod- eling with differential equations and parametric equations, as well as an introduction to linear algebra.

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**x Supplements**

***WileyPLUS*** *WileyPLUS*, Wiley’s digital-learning environment, is loaded with all of the supplements listed on the previous page, and also features the following:

• **Homework management tools**, which easily allow you to assign and grade algorithmic questions, as well as gauge student comprehension.

• **Algorithmic** questions with randomized numeric values and an answer-entry palette for symbolic notation are provided online though *WileyPLUS*. Students can click on “help” buttons for hints, link to the relevant section of the text, show their work or query their instructor using a white board, or see a step-by-step solution (depending on instructor- selecting settings).

• **Interactive Illustrations** can be used in the classroom or computer lab, or for student practice.

• **QuickStart** predesigned reading and homework assignments. Use them as-is or customize them to fit the needs of your classroom.

• The **e-book**, which is an exact version of the print text but also features hyperlinks to questions, definitions, and supplements for quicker and easier support.

• **Guided Online (GO) Tutorial Exercises** that prompt students to build solutions step by step. Rather than simply grading an exercise answer as wrong, GO tutorial problems show students precisely where they are making a mistake.

• **Are You Ready?** quizzes gauge student mastery of chapter concepts and techniques and provide feedback on areas that require further attention.

• **Algebra and Trigonometry Refresher** quizzes provide students with an opportunity to brush up on the material necessary to master calculus, as well as to determine areas that require further review.

***WileyPLUS.*** Learn more at www.wileyplus.com.

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**xviii The Roots of Calculus**

**THE ROOTS OF CALCULUS**

Today’s exciting applications of calculus have roots that can be traced to the work of the Greek mathematician Archimedes, but the actual discovery of the fundamental principles of cal- culus was made independently by Isaac Newton (English) and Gottfried Leibniz (German) in the late seventeenth century. The work of Newton and Leibniz was motivated by four major classes of scientific and mathematical problems of the time:

• Find the tangent line to a general curve at a given point.

• Find the area of a general region, the length of a general curve, and the volume of a general solid.

• Find the maximum or minimum value of a quantity—for example, the maximum and minimum distances of a planet from the Sun, or the maximum range attainable for a pro- jectile by varying its angle of fire.

• Given a formula for the distance traveled by a body in any specified amount of time, find the velocity and acceleration of the body at any instant. Conversely, given a formula that

specifies the acceleration of velocity at any instant, find the distance traveled by the body in a specified period of time.

Newton and Leibniz found a fundamental relationship be- tween the problem of finding a tangent line to a curve and the problem of determining the area of a region. Their real- ization of this connection is considered to be the “discovery of calculus.” Though Newton saw how these two problems are related ten years before Leibniz did, Leibniz published his work twenty years before Newton. This situation led to a stormy debate over who was the rightful discoverer of calculus. The debate engulfed Europe for half a century, with the scien- tists of the European continent supporting Leibniz and those from England supporting Newton. The conflict was extremely unfortunate because Newton’s inferior notation badly ham- pered scientific development in England, and the Continent in turn lost the benefit of Newton’s discoveries in astronomy and physics for nearly fifty years. In spite of it all, Newton and Leibniz were sincere admirers of each other’s work.

**ISAAC NEWTON (1642–1727)**

Newton was born in the village of Woolsthorpe, England. His father died before he was born and his mother raised him on the family farm. As a youth he showed little evidence of his later brilliance, except for an unusual talent with mechanical devices—he apparently built a working water clock and a toy flour mill powered by a mouse. In 1661 he entered Trinity College in Cambridge with a deficiency in geometry. Fortunately, Newton caught the eye of Isaac Barrow, a gifted mathematician and teacher. Under Barrow’s guidance Newton immersed himself in mathematics and science, but he graduated without any special distinction. Because the bubonic plague was spreading rapidly through London, Newton returned to his home in Woolsthorpe and stayed there during the years of 1665 and 1666. In those two momentous years the entire framework of modern science was miraculously created in Newton’s mind. He discovered calculus, recognized the underlying principles of planetary motion and gravity, and determined that “white” sunlight was composed of all colors, red to violet. For whatever reasons he kept his discoveries to himself. In 1667 he returned to Cambridge to obtain his Master’s degree and upon graduation became a teacher at Trinity. Then in 1669 Newton succeeded his teacher, Isaac Barrow, to the Lucasian chair of mathematics at Trinity, one of the most honored chairs of

[*Image: Public domain image from http://commons.wikimedia.org/ wiki/File:Hw-newton.jpg. Image provided courtesy of the University of Texas Libraries, The University of Texas at Austin.*]

mathematics in the world.

Thereafter, brilliant discoveries flowed from Newton steadily. He formulated the law of gravitation and used it to explain the motion of the moon, the planets, and the tides; he formulated basic theories of light, thermodynamics, and hydrodynamics; and he devised and constructed the first modern reflecting telescope. Throughout his life Newton was hesitant to publish his major discoveries, revealing them only to a select

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**GEOMETRY FORMULAS**

*A* = area, *S* = lateral surface area, *V* = volume, *h* = height, *B* = area of base, *r* = radius, *l* = slant height, *C* = circumference, *s* = arc length

Parallelogram

Triangle Trapezoid Circle Sector

*a*

*hh*

*hr us bbb*

*A = bh*

*A =* 12

*bh A =* 12(*a* + *b*)*h A = pr*2*, C* = 2*pr*

*A =* (*u* in 12

*r*radians)

2*u, s = ru*

Right Circular Cylinder Right Circular Cone Any Cylinder or Prism with Parallel Bases Sphere

*h*

*r*

*V = pr*2*h, S* = 2*prh* 13

*V = Bh*

*V = pr*3*, S =* 4*pr*2

**ALGEBRA FORMULAS**

**THE QUADRATIC**

**FORMULA THE BINOMIAL FORMULA**

The solutions of the quadratic equation *ax*2 + *bx* + *c* = 0 are *x* = −*b* ± √*b*2 − 4*ac*

2*a*

*r*

*h*

*l*

*h*

*h*

*r*

*r*

*B*

*B*

*V = pr*2*h, S = prl*

43

*(x* + *y)n* = *xn* + *nxn*−1*y* + *n(n* − 1 · 2 1*)*

*xn*−2*y*2 + *n(n* − 1 · 1*)(n* 2 · 3 − 2*)*

*xn*−3*y*3 +···+ *nxyn*−1 + *yn*

*(x* − *y)n* = *xn* − *nxn*−1*y* + *n(n* − 1 · 2 1*)*

*xn*−2*y*2 − *n(n* − 1 · 1*)(n* 2 · 3 − 2*)*

*xn*−3*y*3 +···± *nxyn*−1 ∓ *yn*

**TABLE OF INTEGRALS**

**BASIC FUNCTIONS**

**1.**

**2.**

∫ ∫ *un du* = *n un*+1

+ 1 + *C duu* = ln |*u*| + *C*

**10.**

∫

*au du* = ln *au*

*a* + *C*

**11.**

∫

ln *udu* = *u* ln *u* − *u* + *C* ∫

∫ **3.**

*eu du* = *eu* + *C*

**12.**

cot *udu* = ln |sin *u*| + *C* ∫

∫ **4.**

sin *udu* = − cos *u* + *C*

**13.** sec*udu* = = **5.**

∫

cos *udu* = sin*u* + *C*

ln|sec*u* ln|tan ( 14 + *π* tan + 12*u*| *u*) + | + *C*

*C*

**6.**

**14.**

∫ ∫

csc*udu* = ln|csc*u* − cot *u*| + *C* tan *udu* = ln|sec*u*| + *C* ∫

= ln|tan 12*u*| + *C*

**7.**

sin−1 *udu* = *u* sin−1 *u* + √1 − *u*2 + *C*

**15.**

∫

cot−1 *udu* = *u* cot−1 *u* + ln√1 + *u*2 + *C*

**8.**

∫

cos−1 *udu* = *u* cos−1 *u* − √1 − *u*2 + *C*

**16.**

∫

sec−1 *udu* = *u* sec−1 *u* − ln|*u* + √*u*2 − 1| + *C*

**9.**

**17.**

∫ csc−1 *udu* = *u* csc−1 *u* + ln|*u* + ∫

tan−1 *udu* = *u* tan−1 *u* − ln √*u*2 − 1| + *C*

√1 + *u*2 + *C*

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**RECIPROCALS OF BASIC FUNCTIONS**

**18.**

∫ 1 ± 1

sin*u du* = tan*u* ∓ sec*u* + *C*

**19.**

∫ ∫ **22.**

1 1 ± cot *u du* = 12*(u* ∓ ln |sin*u* ± cos*u*|*)* + *C*

1 ± **23.**

1 cos*u du* = −cot *u* ± csc*u* + *C*

**20.**

∫ ∫ 1

1 ± tan *u du* = 12 *(u* ± ln|cos *u* ± sin *u*|*)* + *C*

1 ± 1

sec*u du* = *u* + cot *u* ∓ csc *u* + *C*

**24.**

**21.**

∫ ∫ sin *u* 1

cos *u du* = ln |tan*u*| + *C*

1 ± 1

csc*u du* = *u* − tan*u* ± sec*u* + *C*

**25.**

∫ 1

1 ± *eu du* = *u* − ln*(*1 ± *eu)* + *C*

**POWERS OF TRIGONOMETRIC FUNCTIONS**

**26.**

∫

sin2 *udu* = 12 *u* − 14 sin 2*u* + *C*

**27.**

∫ **32.**

cot2 *udu* = −cot *u* − *u* + *C* ∫

cos2 *udu* = 12 *u* + 14 sin 2*u* + *C*

**33.**

**28.**

∫

sec2 *udu* = tan *u* + *C* ∫

tan2 *udu* = tan *u* − *u* + *C*

**34.**

**29.**

∫

csc2 *udu* = − cot *u* + *C* ∫

sin*n udu* = − *n* 1sin*n*−1 *u* cos *u* + *n* − *n*

1

∫

sin*n*−2 *udu*

**35.**

**30.**

∫

cot*n udu* = − *n* − 1

1 cot*n*−1 *u* −

∫

cot*n*−2 *udu* ∫

cos*n udu* = *n* 1cos*n*−1 *u* sin *u* + *n* − *n*

1

∫

cos*n*−2 *udu*

**36.**

**31.**

∫

sec*n udu* = *n* − 1

1 sec*n*−2 *u* tan*u* + *n n* − − 2 1

∫

sec*n*−2 *udu* ∫

tan*n udu* = *n* − 1

1 tan*n*−1 *u* −

∫

tan*n*−2 *udu*

**37.**

∫

csc*n udu* = − *n* − 1

1 csc*n*−2 *u* cot *u* + *n n* − − 2 1

∫

csc*n*−2 *udu*

**PRODUCTS OF TRIGONOMETRIC FUNCTIONS**

**38.**

∫

sin *mu* sin *nudu* = −sin*(m* 2*(m* + + *n) n)u*

+ sin*(m* 2*(m* − − *n) n)u*

+ *C*

**39.**

**40.** ∫

cos *mu* cos *nudu* = sin*(m* 2*(m* + + *n) n)u*

+ sin*(m* 2*(m* − − *n) n)u*

+ *C*

∫

sin *mu* cos*nudu* = −cos*(m* 2*(m* + + *n) n)u*

− cos*(m* 2*(m* − − *n) n)u*

+ *C*

**41.**

∫

sin*m u* cos*n udu* = −sin*m*−1 *m u* + cos*n n*+1 *u*

+ *m m* + − *n* 1

∫

sin*m*−2 *u*cos*n udu*

= sin*m*+1 *m u* + cos*n n*−1 *u*

+ *m n* − + 1 *n*

∫

sin*m u* cos*n*−2 *udu*

**PRODUCTS OF TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS**

**42.**

∫

*eau* sin*budu* = *a*2 *e*+ *au*

*b*2 *(a* sin*bu* − *b* cos*bu)* + *C* **43.**

∫

*eau* cos*budu* = *a*2 *e*+ *au*

*b*2 *(a* cos*bu* + *b*sin*bu)* + *C*

**POWERS OF *u* MULTIPLYING OR DIVIDING BASIC FUNCTIONS**

**44.**

∫

*u* sin *udu* = sin*u* − *u* cos*u* + *C*

**45.**

∫ **51.**

*ueu du* = *eu(u* − 1*)* + *C* ∫

*u* cos *udu* = cos*u* + *u* sin*u* + *C*

**52.**

**46.**

∫

*uneu du* = *uneu* − *n*∫

*un*−1*eu du* ∫

*u*2 sin *udu* = 2*u* sin *u* + *(*2 − *u*2*)*cos*u* + *C*

**53.**

**47.**

∫

*unau du* = *u*ln*a nau*

− ln *n*

*a*

∫

*un*−1*au du* + *C* ∫

*u*2 cos *udu* = 2*u* cos *u* + *(u*2 − 2*)*sin*u* + *C*

**54.**

**48.**

∫ *eu du*

*un* = − *(n* − *eu*

1*)un*−1 + *n* − 1

1

∫ *eu du*

∫

∫

*un*−1 *un* sin *udu* = −*un* cos*u* + *n*

*un*−1 cos *udu*

**55.**

**49.**

∫ *au du*

*un* = − *(n* − *au*

1*)un*−1 + *n* ln*a*

− 1

∫ *au du*

∫

∫

*un*−1 *un* cos *udu* = *un* sin *u* − *nun*−1 sin *udu*

**56.**

**50.**

∫ *u du*

ln *u* = ln|ln *u*| + *C*

**POLYNOMIALS MULTIPLYING BASIC FUNCTIONS**

**57.**

∫

*un* ln *udu* = *(n u*+ *n*+1

1*)*2 [*(n* + 1*)* ln *u* − 1] + *C*

∫

*p(u)eau du* = 1*a p(u)eau* − *a*12 *p (u)eau* + *a*13 *p (u)eau* −··· [signs alternate: +−+−···]

**58.**

∫

*p(u)* sin *audu* = −*a* 1*p(u)* cos*au* + *a*12 *p (u)* sin *au* + *a*13 *p (u)* cos*au* −··· [signs alternate in pairs after first term: ++−−++−−···]

**59.**

∫

*p(u)* cos *audu* = 1*a p(u)* sin *au* + *a*12 *p (u)* cos*au* − *a*13 *p (u)* sin*au* −··· [signs alternate in pairs: ++−−++−−···]

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**FOR THE STUDENT**

Calculus provides a way of viewing and analyzing the physi- cal world. As with all mathematics courses, calculus involves equations and formulas. However, if you successfully learn to use all the formulas and solve all of the problems in the text but do not master the underlying *ideas*, you will have missed the most important part of calculus. If you master these ideas, you will have a widely applicable tool that goes far beyond textbook exercises.

Before starting your studies, you may find it helpful to leaf through this text to get a general feeling for its different parts:

■ The opening page of each chapter gives you an overview of what that chapter is about, and the opening page of each section within a chapter gives you an overview of what that section is about. To help you locate specific information, sections are subdivided into topics that are marked with a box like this .

■ Each section ends with a set of exercises. The answers to most odd-numbered exercises appear in the back of the book. If you find that your answer to an exercise does not match that in the back of the book, do not assume immedi- ately that yours is incorrect—there may be more than one way √2*/*2 to and express the text the answer. answer is For 1*/*example, √2 if your answer is , then both are correct since your answer can be obtained by “rationalizing” the text answer. In general, if your answer does not match that in the text, then your best first step is to look for an algebraic manipulation or a trigonometric identity that might help you determine if the two answers are equivalent. If the answer is in the form of a decimal approximation, then your answer might differ from that in the text because of a difference in the number of decimal places used in the computations.

■ The section exercises include regular exercises and four special categories: *Quick Check*, *Focus on Concepts*, *True/False*, and *Writing*.

• The*Quick Check* exercises are intended to give you quick feedback on whether you understand the key ideas in the section; they involve relatively little computation, and have answers provided at the end of the exercise set.

• The*Focus on Concepts*exercises, as their name suggests, key in on the main ideas in the section.

• *True/False* exercises focus on key ideas in a different way. You must decide whether the statement is true in*all possible circumstances*, in which case you would declare it to be “true,” or whether there are some circumstances in which it is not true, in which case you would declare it to be “false.” In each such exercise you are asked to “Explain your answer.” You might do this by noting a theorem in the text that shows the statement to be true or

by finding a particular example in which the statement is not true.

• *Writing* exercises are intended to test your ability to ex- plain mathematical ideas in words rather than relying solely on numbers and symbols. All exercises requiring writing should be answered in complete, correctly punc- tuated logical sentences—not with fragmented phrases and formulas.

■ Each chapter ends with two additional sets of exercises: *Chapter Review Exercises*, which, as the name suggests, is a select set of exercises that provide a review of the main concepts and techniques in the chapter, and *Making Con- nections*, in which exercises require you to draw on and combine various ideas developed throughout the chapter.

■ Your instructor may choose to incorporate technology in your calculus course. Exercises whose solution involves the use of some kind of technology are tagged with icons to alert you and your instructor. Those exercises tagged with the icon require graphing technology—either a graphing calculator or a computer program that can graph equations. Those exercises tagged with the icon **C** require a com- puter algebra system (CAS) such as *Mathematica*, *Maple*, or available on some graphing calculators.

■ At the end of the text you will find a set of four appen- dices covering various topics such as a detailed review of trigonometry and graphing techniques using technology. Inside the front and back covers of the text you will find endpapers that contain useful formulas.

■ The ideas in this text were created by real people with in- teresting personalities and backgrounds. Pictures and bio- graphical sketches of many of these people appear through- out the book.

■ Notes in the margin are intended to clarify or comment on important points in the text.

**A Word of Encouragement**

As you work your way through this text you will find some ideas that you understand immediately, some that you don’t understand until you have read them several times, and others that you do not seem to understand, even after several readings. Do not become discouraged—some ideas are intrinsically dif- ficult and take time to “percolate.” You may well find that a hard idea becomes clear later when you least expect it.

**Web Sites for this Text**

**www.antontextbooks.com www.wiley.com***/***go***/***global***/***anton**

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0

BEFORE CALCULUS

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*The development of calculus in the*

One of the important themes in calculus is the analysis of relationships between physical or *seventeenth and eighteenth*

mathematical quantities. Such relationships can be described in terms of graphs, formulas, *centuries was motivated by the need*

numerical data, or words. In this chapter we will develop the concept of a “function,” which is *to understand physical phenomena*

the basic idea that underlies almost all mathematical and physical relationships, regardless of *such as the tides, the phases of the*

the form in which they are expressed. We will study properties of some of the most basic *moon, the nature of light, and*

functions that occur in calculus, including polynomials, trigonometric functions, inverse *gravity.*

trigonometric functions, exponential functions, and logarithmic functions.

**0.1 FUNCTIONS**

*In this section we will define and develop the concept of a “function,” which is the basic mathematical object that scientists and mathematicians use to describe relationships between variable quantities. Functions play a central role in calculus and its applications.*

**DEFINITION OF A FUNCTION** Many scientific laws and engineering principles describe how one quantity depends on another. This idea was formalized in 1673 by Gottfried Wilhelm Leibniz (see p. xx) who coined the term *function* to indicate the dependence of one quantity on another, as described in the following definition.

**0.1.1 definition** If a variable *y* depends on a variable *x* in such a way that each value of *x* determines exactly one value of *y*, then we say that ***y is a function of x***.

Four common methods for representing functions are:

• Numerically by tables • Geometrically by graphs

• Algebraically by formulas • Verbally

**1**

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**2 Chapter 0 / Before Calculus**

**Table 0.1.1**

The method of representation often depends on how the function arises. For example: INDIANAPOLIS 500 qUALIFYING SPEEDS YEAR *t* SPEED *S*

• Table 0.1.1 shows the top qualifying speed *S* for the Indianapolis 500 auto race as a

(mi/h) 1994 1995 1996 1997

function of the year *t*. There is exactly one value of *S* for each value of *t*.

• Figure 0.1.1 is a graphical record of an earthquake recorded on a seismograph. The graph describes the deflection *D* of the seismograph needle as a function of the time *T* elapsed since the wave left the earthquake’s epicenter. There is exactly one value of *D* for each value of *T* . 1998 1999 2000 2001 2002 2003

• Some of the most familiar functions arise from formulas; for example, the formula *C* = 2*πr* expresses the circumference *C* of a circle as a function of its radius *r*. There is exactly one value of *C* for each value of *r*.

• Sometimes functions are described in words. For example, Isaac Newton’s Law of Universal Gravitation is often stated as follows: The gravitational force of attraction 2004

between two bodies in the Universe is directly proportional to the product of their 2005

masses and inversely proportional to the square of the distance between them. This 2006

is the verbal description of the formula 2007 2008 2009

*F* = *Gmr*1*m*2

2010 2011

228.011 231.604 233.100 218.263 223.503 225.179 223.471 226.037 231.342 231.725 222.024 227.598 228.985 225.817 226.366

2

224.864 227.970

in which *F* is the force of attraction, *m*1 and *m*2 are the masses, *r* is the distance be- 227.472

tween them, and*G*is a constant. If the masses are constant, then the verbal description defines *F* as a function of *r*. There is exactly one value of *F* for each value of *r*.

*D*

Time of Arrival of Arrival of earthquake *P*-waves

*S*-waves shock

9.4

Surface waves 11.8

minutes minutes

0 Time in minutes

10 20 30 40 50 60 70 80 *T*

**Figure 0.1.1**

In the mid-eighteenth century the Swiss mathematician Leonhard Euler (pronounced

*f*

Computer Program Input *x* Output *y*

“oiler”) conceived the idea of denoting functions by letters of the alphabet, thereby making it possible to refer to functions without stating specific formulas, graphs, or tables. To understand Euler’s idea, think of a function as a computer program that takes an *input x*, operates on it in some way, and produces exactly one *output y*. The computer program is an object in its own right, so we can give it a name, say *f*. Thus, the function *f* (the computer program) associates a unique output *y* with each input *x* (Figure 0.1.2). This suggests the **Figure 0.1.2** following definition.

) sdnuop(225 200 175 150

**0.1.2 definition** A ***function*** *f* is a rule that associates a unique output with each input. If the input is denoted by *x*, then the output is denoted by *f (x)* (read “*f* of *x*”). 125 100 75 50In this definition the term *unique* means “exactly one.” Thus, a function cannot assign

10 15 20 25 30

two different outputs to the same input. For example, Figure 0.1.3 shows a plot of weight Age *A* (years) **Figure 0.1.3**

versus age for a random sample of 100 college students. This plot does *not* describe *W* as a function of *A* because there are some values of *A* with more than one corresponding

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**0.1 Functions 3**

value of *W*. This is to be expected, since two people with the same age can have different weights.

**INDEPENDENT AND DEPENDENT VARIABLES** For a given input *x*, the output of a function *f* is called the ***value*** of *f* at *x* or the ***image*** of *x* under *f*. Sometimes we will want to denote the output by a single letter, say *y*, and write

*y* = *f(x)*

This equation expresses *y* as a function of *x*; the variable *x* is called the ***independent variable*** (or ***argument***) of *f* , and the variable *y* is called the ***dependent variable*** of *f*. This terminology is intended to suggest that*x* is free to vary, but that once *x* has a specific value a corresponding value of *y* is determined. For now we will only consider functions in which the independent and dependent variables are real numbers, in which case we say that *f* is a ***real-valued function of a real variable***. Later, we will consider other kinds of functions.

**Table 0.1.2 Example 1** Table 0.1.2 describes a functional relationship *y* = *f (x)* for which 03 *xf(*0*)* = 3 *f* associates *y* = 3 with *x* = 0.

*y*

*f(*1*)* = 4 *f* associates *y* = 4 with *x* = 1.

*f(*2*)* = −1 *f* associates *y* = −1 with *x* = 2.

*f(*3*)* = 6 *f* associates *y* = 6 with *x* = 3.

**Example 2** The equation

*y* = 3*x*2 − 4*x* + 2

has the form *y* = *f(x)* in which the function *f* is given by the formula

*f(x)* = 3*x*2 − 4*x* + 2 1234 −1

6

**Leonhard Euler (1707–1783)** Euler was probably the most prolific mathematician who ever lived. It has been said that “Euler wrote mathematics as effortlessly as most men breathe.” He was born in Basel, Switzerland, and was the son of a Protestant minister who had himself studied mathematics. Euler’s genius developed early. He attended the University of Basel, where by age 16 he obtained both a Bachelor of Arts degree and a Master’s degree in philosophy. While at Basel, Euler had the good fortune to be tutored one day a week in mathematics by a distinguished mathematician, Johann Bernoulli. At the urging of his father, Euler then began to study theology. The lure of mathematics was too great, however, and by age 18 Euler had begun to do mathematical research. Nevertheless, the influence of his father and his theological studies remained, and throughout his life Euler was a deeply religious, unaffected person. At various times Euler taught at St. Petersburg Academy of Sciences (in Rus- sia), the University of Basel, and the Berlin Academy of Sciences. Euler’s energy and capacity for work were virtually boundless. His collected works form more than 100 quarto-sized volumes and it is believed that much of his work has been lost. What is particularly

astonishing is that Euler was blind for the last 17 years of his life, and this was one of his most productive periods! Euler’s flawless memory was phenomenal. Early in his life he memorized the entire *Aeneid* by Virgil, and at age 70 he could not only recite the entire work but could also state the first and last sentence on each page of the book from which he memorized the work. His ability to solve problems in his head was beyond belief. He worked out in his head major problems of lunar motion that baffled Isaac Newton and once did a complicated calculation in his head to settle an argument between two students whose computations differed in the fiftieth decimal place.

Following the development of calculus by Leibniz and Newton, results in mathematics developed rapidly in a disorganized way. Eu- ler’s genius gave coherence to the mathematical landscape. He was the first mathematician to bring the full power of calculus to bear on problems from physics. He made major contributions to virtu- ally every branch of mathematics as well as to the theory of optics, planetary motion, electricity, magnetism, and general mechanics.

[*Image: http://commons.wikimedia.org/wiki/File:Leonhard\_Euler\_by\_Handmann\_.png*]

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**4 Chapter 0 / Before Calculus**

For each input *x*, the corresponding output *y* is obtained by substituting *x* in this formula. For example,

*f(*0*)* = 3*(*0*)*2 − 4*(*0*)* + 2 = 2 *f* associates *y* = 2 with *x* = 0*.*

*f(*−1*.*7*)* = 3*(*−1*.*7*)*2 − 4*(*−1*.*7*)* + 2 = 17*.*47 *f* associates *y* = 17*.*47 with *x* = −1*.*7*. f(*√2 *)* = 3*(*√2 *)*2 − 4√2 + 2 = 8 − 4√2 *f* associates *y* = 8 − 4√2 with *x* = √2*.*

**GRAPHS OF FUNCTIONS** If *f* is a real-valued function of a real variable, then the ***graph*** of *f* in the *xy*-plane is Figure 0.1.4 shows only portions of the

defined to be the graph of the equation *y* = *f(x)*. For example, the graph of the function graphs. Where appropriate, and unless indicated otherwise, it is understood that graphs shown in this text extend indefinitely beyond the boundaries of

*f(x)* = *x* is the graph of the equation *y* = *x*, shown in Figure 0.1.4. That figure also shows the graphs of some other basic functions that may already be familiar to you. In Appendix A we discuss techniques for graphing functions using graphing technology.

the displayed figure.

4

*y y* = *x y* = *x*2 *y* = *x*3

3210*x* −1−2−3−4−4 −3 −2 −1 0 1 2 3 4 *y* = 1/*x*

*y*

8

*y*

6420*x* −2−4−6−8−8 −6 −4 −2 20 4 6 8 4 3210*x* −1−2−3−4−5−4−3−2−1 10 2 3 4 5 7

6543210*x*

−1−3 −2 −1 10 2 3 *y* 4

*y*

*y* = √*x* 4

*y*

*y* = √*x* 3

3232Since √*x* is imaginary for negative val- ues graph of of *x*, *y* there = √*x* are in no points 10*x*

10*x* the region −1−2−3−4−8 −6 −4 −2 20 4 6 8 on the

where *x <* 0.

−1−2−3−4−1 0 1 2 3 4 5 6 7 8 9 **Figure 0.1.4**

*y*

Graphs can provide valuable visual information about a function. For example, since the graph of a function *f* in the *xy*-plane is the graph of the equation *y* = *f(x)*, the points *f*(*x*) (*x*, *f*(*x*)) on the graph of *f* are of the form *(x,f(x))*; that is, *the y*-*coordinate of a point on the graph of f is the value of f at the corresponding x-coordinate* (Figure 0.1.5). The values of *x y* = *f*(*x*)

for which *f(x)* = 0 are the *x*-coordinates of the points where the graph of *f* intersects the *x*-axis (Figure 0.1.6). These values are called the ***zeros*** of *f*, the ***roots*** of *f(x)* = 0, or the *x*

***x-intercepts*** of the graph of *y* = *f(x)*.

**Figure 0.1.5** The *y*-coordinate of a point on the graph of *y* = *f(x)* is the value of *f* at the corresponding *x*-coordinate.

*x*

**THE VERTICAL LINE TEST** Not every curve in the *xy*-plane is the graph of a function. For example, consider the curve in Figure 0.1.7, which is cut at two distinct points, *(a, b)* and *(a, c)*, by a vertical line. This curve cannot be the graph of *y* = *f(x)* for any function *f* ; otherwise, we would have

*f(a)* = *b* and *f(a)* = *c*

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**0.1 Functions 5**

*y*

which is impossible, since *f* cannot assign two different values to *a*. Thus, there is no *y* = *f*(*x*)

*x x*1 0 *x*2 *x*3

**Figure 0.1.6** *f* has zeros at *x*1, 0, *x*2, and *x*3.

function *f* whose graph is the given curve. This illustrates the following general result, which we will call the ***vertical line test***.

**0.1.3 the vertical line test** *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*

*y*

**Example 3** The graph of the equation

*x*2 + *y*2 = 25

is a circle of radius 5 centered at the origin and hence there are vertical lines that cut the graph more than once (Figure 0.1.8). Thus this equation does not define *y* as a function of *x*. (*a*, *c*)

(*a*, *b*)

*x a***Figure 0.1.7** This curve cannot be the graph of a function.

6

*x* −6 6

−6**THE ABSOLUTE VALUE FUNCTION**

Recall that the ***absolute value*** or ***magnitude*** of a real number *x* is defined by

|*x*| =

{ *x, x* ≥ 0 −*x, x <* 0

The effect of taking the absolute value of a number is to strip away the minus sign if the

Symbols such as +*x* and −*x* are de- ceptive, since it is tempting to conclude that +*x* is positive and −*x* is negative. However, this need not be so, since *x* itself can be positive or negative. For example, if *x* is negative, say *x* = −3, then −*x* = 3 is positive and +*x* = −3 is negative.

number is negative and to leave |5| the = 5*,* number ∣∣− 47unchanged ∣∣ = 47*,* if it is nonnegative. Thus,

|0| = 0

A more detailed discussion of the properties of absolute value is given in Web Appendix F. However, for convenience we provide the following summary of its algebraic properties.

**0.1.4 properties of absolute value** *If a and b are real numbers, then*

(*a*) |−*a*|=|*a*| A number and its negative have the same absolute value. *y*

(*b*) |*ab*|=|*a*||*b*| The absolute value of a product is the product of the absolute values. *x*2 + *y*2 = 25

(*c*) |*a/b*|=|*a*|*/*|*b*|*,b* = 0 The absolute value of a ratio is the ratio of the absolute values.

(*d*) |*a* + *b*|≤|*a*|+|*b*| The ***triangle inequality***

The graph of the function *f(x)* = |*x*| can be obtained by graphing the two parts of the equation

*y* = **Figure 0.1.8**

{ *x, x* ≥ 0 −*x, x <* 0

separately. Combining the two parts produces the V-shaped graph in Figure 0.1.9.

Absolute values have important relationships to square roots. To see why this is so, recall from negative. algebra By definition, that every positive the symbol real √number *x* denotes *x* has the **WARNING**

two square roots, one positive and one To must denote write the −negative √*x*. *positive* square root of *x*. positive square root square root you For of 9 example, is √9 = the 3, is take whereas −√of 9 writing = the −3. negative √(Do 9 = not ±3.)

square make root the mis- of 9

true Care thatmust √*x*2 be = exercised *x*. in simplifying expressions of the form √*x*2, since it is*not* always This equation is correct if *x* is nonnegative, but it is false if *x* is negative. For example, if *x* = −4, then√*x*2 = √*(*−4*)*2 = √16 = 4 = *x*

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**6 Chapter 0 / Before Calculus**

**TECHNOLOGY MASTERY**

A statement that is correct for all real values of *x* is

Verify show that (1) by the using equations a graphing *y* = √utility *x*2 and to

√*x*2 = |*x*| (1)

*y* = |*x*| have the same graph.

**PIECEWISE-DEFINED FUNCTIONS**

5 4*y y* = |*x*|

The absolute value function *f(x)* = |*x*| is an example of a function that is defined***piecewise*** in the sense that the formula for *f* changes, depending on the value of *x*. 3210**Example 4** Sketch the graph of the function defined piecewise by the formula

*x*

−1−2*f(x)* = −3−5 −4 −3 −2 −1 10 2 3 4 5 **Figure 0.1.9**

⎧⎪⎨⎪⎩0*,* √*x,* 1 − *x*2*, x* ≤ −1 −1 *<x< x* ≥ 1

1

***Solution.*** The formula for *f* changes at the points *x* = −1 and *x* = 1. (We call these the *y*

***breakpoints*** for the formula.) A good procedure for graphing functions defined piecewise 2

is to graph the function separately over the open intervals determined by the breakpoints, 1and then graph *f* graph is the horizontal at the breakpoints themselves. For the function *f* ray*y* = 0 on the interval *(*− *,*−1], it is the semicircle in this *y* example = √1 − the *x*2 on the interval *(*−1*,*1*)*, and it is the ray *y* = *x* on the interval [1*,*+ *)*. The formula for *f*

*x*

specifies that the equation *y* = 0 applies at the breakpoint −1 [so *y* = *f(*−1*)* = 0], and it

−2 −1 1 2

specifies that the equation *y* = *x* applies at the breakpoint 1 [so *y* = *f(*1*)* = 1]. The graph

**Figure 0.1.10**

of *f* is shown in Figure 0.1.10.

**REMARK** In Figure 0.1.10 the solid dot and open circle at the breakpoint *x* = 1 serve to emphasize that the point on the graph lies on the ray and not the semicircle. There is no ambiguity at the breakpoint *x* = −1 because the two parts of the graph join together continuously there.

**Example 5** Increasing the speed at which air moves over a person’s skin increases

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*The wind chill index measures the sensation of coldness that we feel from the combined effect of temperature and wind speed.*

the rate of moisture evaporation and makes the person feel cooler. (This is why we fan ourselves in hot weather.) The ***wind chill index*** is the temperature at a wind speed of 4 mi*/*h that would produce the same sensation on exposed skin as the current temperature and wind speed combination. An empirical formula (i.e., a formula based on experimental data) for the wind chill index *W* at 32◦F for a wind speed of *v* mi*/*h is

*W* =

{32*,* 0 ≤ *v* ≤ 3

55*.*628 − 22*.*07*v*0*.*16*,* 3 *< v*

A computer-generated graph of *W(v)* is shown in Figure 0.1.11.

**Figure 0.1.11** Wind chill versus wind speed at 32◦F

35 3025l lihcd niW201510500 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75

Wind speed *v* (mi/h)

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**0.1 Functions 7**

**DOMAIN AND RANGE** If*x* and*y* are related by the equation*y* = *f(x)*, then the set of all allowable inputs (*x*-values) is called the ***domain*** of *f*, and the set of outputs (*y*-values) that result when *x* varies over the domain is called the ***range*** of *f*. For example, if *f* is the function defined by the table in Example 1, then the domain is the set {0*,*1*,*2*,*3} and the range is the set {−1*,*3*,*4*,*6}.

Sometimes physical or geometric considerations impose restrictions on the allowable inputs of a function. For example, if *y* denotes the area of a square of side *x*, then these variables are related by the equation *y* = *x*2. Although this equation produces a unique value of *y* for every real number *x*, the fact that lengths must be nonnegative imposes the requirement that *x* ≥ 0. One might argue that a physical square cannot have a side of length zero. However, it is often convenient mathe- matically to allow zero lengths, and we will do so throughout this text where appropriate.

When a function is defined by a mathematical formula, the formula itself may impose restrictions input since on the allowable division by zero inputs. is undefined, For example, and if *y* if*y* = √= *x*, 1*/x*, then then*x* negative = 0 values is not an of allowable *x* are not allowable inputs because they produce imaginary values for *y* and we have agreed to consider only real-valued functions of a real variable. In general, we make the following definition.

**0.1.5 definition** If a real-valued function of a real variable is defined by a formula, and if no domain is stated explicitly, then it is to be understood that the domain consists of all real numbers for which the formula yields a real value. This is called the ***natural domain*** of the function.

The domain and range of a function*f* can be pictured by projecting the graph of*y* = *f(x) y*

onto the coordinate axes as shown in Figure 0.1.12.

e gnaR*y* = *f*(*x*)

*x*

Domain

**Figure 0.1.12** The projection of *y* = *f(x)* on the *x*-axis is the set of allowable *x*-values for *f*, and the projection on the *y*-axis is the set of corresponding *y*-values.

**Example 6** Find the natural domain of

(a) *f(x)* = *x*3 (c) *f(x)* = tan *x* (b) (d) *f(x) f(x)* = = 1*/*[*(x* √*x*2 − − 5*x* 1*)(x* + 6

− 3*)*]

***Solution* (*a*)*.*** The function *f* has real values for all real *x*, so its natural domain is the interval *(*− *,*+ *)*.

***Solution* (*b*)*.*** The function *f* has real values for all real *x*, except *x* = 1 and *x* = 3, where divisions by zero occur. Thus, the natural domain is

{*x* : *x* = 1 and *x* = 3} = *(*− *,*1*)* ∪ *(*1*,*3*)* ∪ *(*3*,*+ *)*

***Solution* (*c*)*.*** Since *f(x)* = tan*x* = sin *x/*cos*x*, the function *f* has real values except where cos*x* = 0, and this occurs when*x* is an odd integer multiple of*π/*2. Thus, the natural domain consists of all real numbers except For a review of trigonometry see Ap- pendix B. *x* = ±*π*2*,*±3*π*2 *,*±5*π*2 *,...*

***Solution* (*d*)*.*** The function *f* has real values, except when the expression inside the radical is negative. Thus the natural domain consists of all real numbers *x* such that

*x*2 − 5*x* + 6 = *(x* − 3*)(x* − 2*)* ≥ 0

This inequality is satisfied if *x* ≤ 2 or *x* ≥ 3 (verify), so the natural domain of *f* is

*(*− *,*2]∪[3*,*+ *)*

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*y y* = *x*2

In some cases we will state the domain explicitly when defining a function. For example, if *f(x)* = *x*2 is the area of a square of side *x*, then we can write

*f(x)* = *x*2*, x* ≥ 0

*x*

to indicate that we take the domain of *f* to be the set of nonnegative real numbers (Fig- ure 0.1.13). *x*

**THE EFFECT OF ALGEBRAIC OPERATIONS ON THE DOMAIN** *y y* = *x*2, *x* ≥ 0

Algebraic expressions are frequently simplified by canceling common factors in the nu- merator and denominator. However, care must be exercised when simplifying formulas for functions in this way, since this process can alter the domain.

**Example 7** The natural domain of the function

**Figure 0.1.13**

*f(x)* = *x*2 − 4

*x* − 2 (2)

consists of all real *x* except *x* = 2. However, if we factor the numerator and then cancel

*y*

the common factor in the numerator and denominator, we obtain

6 54*y* = *x* + 2

*f(x)* = *(x* − *x* 2*)(x* − 2 + 2*)*

= *x* + 2 (3)

32Since the right side of (3) has a value of *f(*2*)* = 4 and *f(*2*)* was undefined in (2), the 1*x*

algebraic simplification has changed the function. Geometrically, the graph of (3) is the

−3−2−1 1 2 3 4 5

line in Figure 0.1.14*a*, whereas the graph of (2) is the same line but with a hole at *x* = 2, since the function is undefined there (Figure 0.1.14*b*). In short, the geometric effect of the (*a*)

algebraic cancellation is to eliminate the hole in the original graph.

−3−2−1 1 2 3 4 5

**Figure 0.1.14**

*y* 6 543*y* = *x*2 − 4 *x* − 2

Sometimes alterations to the domain of a function that result from algebraic simplification are irrelevant to the problem at hand and can be ignored. However, if the domain must be preserved, then one must impose the restrictions on the simplified function explicitly. For 21example, if we wanted to preserve the domain of the function in Example 7, then we would

*x*

have to express the simplified form of the function as

*f(x)* = *x* + 2*, x* = 2 (*b*)

**Example 8** Find the domain and range of

(a) *f(x)* = 2 + √*x* − 1 (b) *f(x)* = *(x* + 1*)/(x* − 1*)*

***Solution*** [1*,*+ [0*,*+ *)*. *)*, **(*a*)*.*** Since no As *x* varies over so the value of domain the interval *f(x)* = is stated 2 + [1*,*+ √*x y*

*y* = 2 + √*x* − 1

explicitly, *)*, the value the domain of √*x* − of 1 − 1 *f* is its natural varies over the domain, 5 432interval varies over the interval [2*,*+ *)*, which is the range of *f* . The domain and range are highlighted in green on the *x*- and *y*-axes in Figure 0.1.15.

1*x*

1 2 3 4 5 6 7 8 9 10

**Figure 0.1.15**

***Solution* (*b*)*.*** The given function *f* is defined for all real *x*, except *x* = 1, so the natural domain of *f* is {*x* : *x* = 1} = *(*− *,*1*)* ∪ *(*1*,*+ *)*

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**0.1 Functions 9**

*y*

5

*y* = *x* + 1 *x* − 1 43To determine the range it will be convenient to introduce a dependent variable

*y* = *x x* + − 1

1 (4) 2Although the set of possible *y*-values is not immediately evident from this equation, the 1−3 −2 −1 −1−2*x*

1 2 3 4 5 6

graph of (4), which is shown in Figure 0.1.16, suggests that the range of *f* consists of all *y*, except *y* = 1. To see that this is so, we solve (4) for *x* in terms of *y*:

*(x* − 1*)y* = *x* + 1 *xy* − *y* = *x* + 1 **Figure 0.1.16**

*xy* − *x* = *y* + 1 *x(y* − 1*)* = *y* + 1 *x* = *y y* + − 1 1 It is now evident from the right side of this equation that *y* = 1 is not in the range; otherwise we would have a division by zero. No other values of *y* are excluded by this equation, so the range of the function *f* is {*y* : *y* = 1} = *(*− *,*1*)* ∪ *(*1*,*+ *)*, which agrees with the result obtained graphically.

**DOMAIN AND RANGE IN APPLIED PROBLEMS** In applications, physical considerations often impose restrictions on the domain and range of a function.

**Example 9** An open box is to be made from a 16-inch by 30-inch piece of card- board by cutting out squares of equal size from the four corners and bending up the sides (Figure 0.1.17*a*).

(a) Let *V* be the volume of the box that results when the squares have sides of length *x*.

Find a formula for *V* as a function of *x*. (b) Find the domain of *V* . (c) Use the graph of *V* given in Figure 0.1.17*c* to estimate the range of *V* . (d) Describe in words what the graph tells you about the volume.

***Solution* (*a*)*.*** As shown in Figure 0.1.17*b*, the resulting box has dimensions 16 − 2*x* by 30 − 2*x* by *x*, so the volume *V (x)* is given by

*V (x)* = *(*16 − 2*x)(*30 − 2*x)x* = 480*x* − 92*x*2 + 4*x*3

*xxx*)

800 *x*

ni(x obf o700 *x*

*x*

*x*

0 1 2 3 4 5 6 7 8 9 Side *x* of square cut (in) (*a*) (*b*) (*c*) **Figure 0.1.17**

*xx*

600 16 in

500

30 in

16 – 2*x*

30 – 2*x*

400 300 200 100

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***Solution* (*b*)*.*** The domain is the set of *x*-values and the range is the set of *V*-values. Because *x* is a length, it must be nonnegative, and because we cannot cut out squares whose sides are more than 8 in long (why?), the *x*-values in the domain must satisfy

0 ≤ *x* ≤ 8

***Solution* (*c*)*.*** From the graph of *V* versus *x* in Figure 0.1.17*c* we estimate that the *V* - values in the range satisfy 0 ≤ *V* ≤ 725

Note that this is an approximation. Later we will show how to find the range exactly.

***Solution* (*d*)*.*** The graph tells us that the box of maximum volume occurs for a value of *x* that is between 3 and 4 and that the maximum volume is approximately 725 in3. The graph also shows that the volume decreases toward zero as *x* gets closer to 0 or 8, which should make sense to you intuitively.

In applications involving time, formulas for functions are often expressed in terms of a variable *t* whose starting value is taken to be *t* = 0.

**Example 10** At 8:05 A.M. a car is clocked at 100 ft*/*s by a radar detector that is positioned at the edge of a straight highway. Assuming that the car maintains a constant speed between 8:05 A.M. and 8:06 A.M., find a function *D(t)* that expresses the distance

Radar Tracking

traveled by the car during that time interval as a function of the time *t*. 6000 5000 4000 3000

***Solution.*** It would be clumsy to use the actual clock time for the variable *t*, so let us agree to use the *elapsed* time in seconds, starting with *t* = 0 at 8:05 A.M. and ending with 2000

*t* = 60 at 8:06 A.M. At each instant, the distance traveled (in ft) is equal to the speed of the 1000

car (in ft*/*s) multiplied by the elapsed time (in s). Thus,

0 10 20 30 40 50 60 8:05 A.M. Time *t* (s) 8:06 A.M.

*D(t)* = 100*t,* 0 ≤ *t* ≤ 60

**Figure 0.1.18**

The graph of *D* versus *t* is shown in Figure 0.1.18.

**ISSUES OF SCALE AND UNITS** *y*

In geometric problems where you want to preserve the “true” shape of a graph, you must use units of equal length on both axes. For example, if you graph a circle in a coordinate system in which 1 unit in the *y*-direction is smaller than 1 unit in the *x*-direction, then the circle will be squashed vertically into an elliptical shape (Figure 0.1.19).

The circle is squashed because 1 unit on the *y*-axis has a smaller length than 1 unit on the *x*-axis.

**Figure 0.1.19**

In applications where the variables on the two axes have unrelated units (say, centimeters on the *y*-axis and seconds on the *x*-axis), then nothing is gained by requiring the units to have equal lengths; choose the lengths to make the graph as clear as possible.

*x*

However, sometimes it is inconvenient or impossible to display a graph using units of equal length. For example, consider the equation

*y* = *x*2

If we want to show the portion of the graph over the interval −3 ≤ *x* ≤ 3, then there is no problem using units of equal length, since *y* only varies from 0 to 9 over that interval. However, if we want to show the portion of the graph over the interval −10 ≤ *x* ≤ 10, then there is a problem keeping the units equal in length, since the value of *y* varies between 0 and 100. In this case the only reasonable way to show all of the graph that occurs over the interval −10 ≤ *x* ≤ 10 is to compress the unit of length along the *y*-axis, as illustrated in Figure 0.1.20.

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**0.1 Functions 11**

*y*

*y*

**Figure 0.1.20** −3 −2 −1 1 2 3

−10 −5 5 10

✔**QUICK CHECK EXERCISES 0.1** (*See page 15 for answers.*)

**1.** Let *f(x)* = √*x* + 1 + 4.

(a) The natural domain of *f* is . (b) *f(*3*)* = (c) *f (t*2 − 1*)* = (d) *f(x)* = 7 if *x* = (e) The range of *f* is . **2.** Line segments in an *xy*-plane form “letters” as depicted.

(a) Ifthe*y*-axis is parallel to the letter I, which of the letters

represent the graph of *y* = *f(x)* for some function *f* ? (b) If the *y*-axis is perpendicular to the letter I, which of the letters represent the graph of *y* = *f(x)* for some function *f*? **3.** The accompanying figure shows the complete graph of

*y* = *f(x)*. (a) The domain of *f* is . (b) The range of *f* is . (c) (d) *f(*−3*) f* ( 12) =

=

(e) The solutions to *x* = .

*f(x)* = −32 are *x* = and

2 1*x*

−3 −2 −1 −1321 −29 87654321*x*

100

80

604020*x*

**4.** The accompanying table gives a 5-day forecast of high and

low temperatures in degrees Fahrenheit (◦F). (a) Suppose that *x* and *y* denote the respective high and low temperature predictions for each of the 5 days. Is *y* a function of *x*? If so, give the domain and range of this function. (b) Supposethat *x* and*y* denote the respective low and high temperature predictions for each of the 5 days. Is *y* a function of *x*? If so, give the domain and range of this function.

MON TUE wED THURS FRI

HIGH 7571657073LOw

52

56

48

50

52

**Table Ex-4**

**5.** Let *l*, *w*, and *A* denote the length, width, and area of a rectangle, respectively, and suppose that the width of the rectangle is half the length. (a) If *l* is expressed as a function of *w*, then *l* = . (b) If *A* is expressed as a function of *l*, then *A* = . (c) If *w* is expressed as a function of *A*, then*w* = .

*y*

**Figure Ex-3**

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**12 Chapter 0 / Before Calculus**

**EXERCISE SET 0.1** Graphing Utility

**1.** Use the accompanying graph to answer the following ques- tions, making reasonable approximations where needed. (a) For what values of *x* is *y* = 1? (b) For what values of *x* is *y* = 3? (c) For what values of *y* is *x* = 3? (d) For what values of *x* is *y* ≤ 0? (e) What are the maximum and minimum values of *y* and

for what values of *x* do they occur?

−3 −2 −1 0 1 2 3

**4.** In each part, compare the natural domains of *f* and *g*.

(a) *f(x)* = *xx* 2 + + 1 *x*

; *g(x)* = *x*

(b) *f(x)* = *x*√*x x* + + 1 √*x*

; *g(x)* = √*x*

3

*y*

**FOCUS ON CONCEPTS**

2**5.** The accompanying graph shows the median income in U.S. households (adjusted for inflation) between 1990 1and 2005. Use the graph to answer the following ques- tions, making reasonable approximations where needed. 0*x*

(a) When was the median income at its maximum value, and what was the median income when that occurred? −1(b) When was the median income at its minimum value, and what was the median income when that occurred? −2(c) The median income was declining during the 2-year period between 2000 and 2002. Was it declining −3more rapidly during the first year or the second year

**Figure Ex-1**

of that period? Explain your reasoning.

**2.** Use the accompanying table to answer the questions posed

in Exercise 1.

−2

5

Median U.S. Household Income in e mocnId lohesuoH. S.Un aideM48 4644421990 1995 2000 2005 Thousands of Constant 2005 Dollars

*xy*

***Source:*** U.S. Census Bureau, August 2006.

**Figure Ex-5**

**6.** Use the median income graph in Exercise 5 to answer the following questions, making reasonable approximations where needed. (a) What was the average yearly growth of median in-

come between 1993 and 1999? (b) The median income was increasing during the 6-year period between 1993 and 1999. Was it increasing more rapidly during the first 3 years or the last 3 years of that period? Explain your reasoning. (c) Consider the statement: “After years of decline, me- dian income this year was finally higher than that of last year.” In what years would this statement have been correct? −1

021 −2

7

3−1

41

50

69

**Table Ex-2**

**3.** In each part of the accompanying figure, determine whether

the graph defines *y* as a function of *x*.

*y*

*y*

(*a*)

(*b*)

*x*

**Figure Ex-3**

*x*

*x*

*y*

(*c*)

*y*

(*d*)

*x*

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**0.1 Functions 13**

**7.** Find *f(*0*),f(*2*),f(*−2*),f(*3*),f(*√2 *)*, and *f(*3*t)*.

(a) *f(x)* = 3*x*2 − 2 (b) *f(x)* =

⎧⎨⎩

2*x, x* 1*, x >* 3 *x* ≤ 3

**14.** A cup of hot coffee sits on a table. You pour in some cool milk and let it sit for an hour. Sketch a rough graph of the temperature of the coffee as a function of time.

**8.** Find *g(*3*), g(*−1*), g(π), g(*−1*.*1*)*, and *g(t*2 − 1*)*.

**15–18** As seen in Example 3, the equation *x*2 + *y*2 = 25 does

(a) *g(x)* = *x x* + − 1

1 (b) *g(x)* =

{√*x* + 1*, x* ≥ 1 3*, x <* 1

not define *y* as a function of *x*. Each graph in these exercises is a portion of the circle *x*2 + *y*2 = 25. In each case, determine whether the graph defines *y* as a function of *x*, and if so, give a

**9–10** Find the natural domain and determine the range of each function. If you have a graphing utility, use it to confirm that your result is consistent with the graph produced by your graph-

formula for *y* in terms of *x*. ■ **15.**

5

ing utility. [*Note:* Set your graphing utility in radian mode when graphing trigonometric functions.] ■

*x* **9.** (a) −5 5

(c) −5*f(x) g(x)* = = √*x x*− 1

2 3 − 3 (b) (d) *F(x) G(x)* = = *x*|*x*|

√*x*2 − 2*x* + 5 (e) *h(x)* = 1 − 1

sin *x* (f ) *H(x)* =

*y* **16.**

*y* 5

*x* −5 5

√*x*2 − 4

−5*x* − 2

**17. 10.** (a) (c) *f(x) g(x)* = = 3 √+ 3 − √*x x* (e) *h(x)* = 3 sin *x* (b) *F(x)* = √4 − *x*2

(d) (f ) *G(x) H(x)* = = *x(*sin 3 + √2

*x)*−2

5

*x*

−5 5

**FOCUS ON CONCEPTS**

**11.** (a) If you had a device that could record the Earth’s pop-

−5ulation continuously, would you expect the graph of population versus time to be a continuous (unbro- ken) curve? Explain what might cause breaks in the curve. (b) Suppose that a hospital patient receives an injection of an antibiotic every 8 hours and that between in- jections the concentration *C* of the antibiotic in the bloodstream decreases as the antibiotic is absorbed by the tissues. What might the graph of *C* versus the elapsed time *t* look like? **12.** (a) If you had a device that could record the tempera- ture of a room continuously over a 24-hour period, would you expect the graph of temperature versus time to be a continuous (unbroken) curve? Explain your reasoning. (b) If you had a computer that could track the number of boxes of cereal on the shelf of a market contin- uously over a 1-week period, would you expect the graph of the number of boxes on the shelf versus time to be a continuous (unbroken) curve? Explain your reasoning. **13.** A boat is bobbing up and down on some gentle waves. Suddenly it gets hit by a large wave and sinks. Sketch a rough graph of the height of the boat above the ocean floor as a function of time.

*y* **18.**

*y* 5

*x* −5 5

−5**19–22 True–False** Determine whether the statement is true or false. Explain your answer. ■ **19.** A curve that crosses the*x*-axis at two different points cannot

be the graph of a function. **20.** The natural domain of a real-valued function defined by a formula consists of all those real numbers for which the formula yields a real value. **21.** The range of the absolute value function is all positive real **22.** numbers.

If *g(x)* = 1*/*√*f(x)*, then the domain of *g* consists of all those real numbers *x* for which *f(x)* = 0. **23.** Use the equation *y* = *x*2 − 6*x* + 8 to answer the following

questions. (a) For what values of *x* is *y* = 0? (b) For what values of *x* is *y* = −10? (c) For what values of *x* is *y* ≥ 0? (d) Does *y* have a minimum value? A maximum value? If **24.** Use so, the find equation them.

*y* = 1 + √*x* to answer the following ques- tions. (a) For what values of *x* is *y* = 4? (b) For what values of *x* is *y* = 0? (c) For what values of *x* is *y* ≥ 6? (*cont.*)

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**14 Chapter 0 / Before Calculus**

(d) Does *y* have a minimum value? A maximum value? If

so, find them. **25.** As shown in the accompanying figure, a pendulum of con- stant length *L* makes an angle *θ* with its vertical position. Express the height *h* as a function of the angle *θ*. **26.** Express the length *L* of a chord of a circle with radius 10 cm as a function of the central angle *θ* (see the accompanying figure).

*L*

*h*

(d) Plot the function in part (b) and estimate the dimensions of the enclosure that minimize the amount of fencing required. **32.** As shown in the accompanying figure, a camera is mounted at a point 3000 ft from the base of a rocket launching pad. The rocket rises vertically when launched, and the camera’s elevation angle is continually adjusted to follow the bottom of the rocket. (a) Express the height *x* as a function of the elevation an-

*u*

*L u*

10 cm

gle *θ*. (b) Find the domain of the function in part (a). (c) Plot the graph of the function in part (a) and use it to estimate the height of the rocket when the elevation an- gle is *π/*4 ≈ 0*.*7854 radian. Compare this estimate to the exact height.

**Figure Ex-25**

**Figure Ex-26**

**27–28** Express the function in piecewise form without using absolute values. [*Suggestion:* It may help to generate the graph of the function.] ■ **27.** (a) *f(x)* = |*x*| + 3*x* + 1 (b) *g(x)* = |*x*|+|*x* − 1| **28.** (a) *f(x)* = 3 + |2*x* − 5| (b) *g(x)* = 3|*x* − 2|−|*x* + 1| **29.** As shown in the accompanying figure, an open box is to be constructed from a rectangular sheet of metal, 8 in by 15

3000 ft

in, by cutting out squares with sides of length *x* from each corner and bending up the sides. (a) Express the volume *V* as a function of *x*. (b) Find the domain of *V* . (c) Plot the graph of the function *V* obtained in part (a) and

estimate the range of this function. (d) In words, describe how the volume *V* varies with *x*, and discuss how one might construct boxes of maximum volume.

*xx*

Rocket

*x*

*u*

Camera

**Figure Ex-32**

**33.** A soup company wants to manufacture a can in the shape of a right circular cylinder that will hold 500 cm3 of liquid. The material for the top and bottom costs 0.02 cent*/*cm2, and the material for the sides costs 0.01 cent*/*cm2. (a) Estimate the radius *r* and the height *h* of the can that costs the least to manufacture. [*Suggestion:* Express the cost *C* in terms of *r*.] (b) Suppose that the tops and bottoms of radius *r* are *xx*punched out from square sheets with sides of length 2*r* and the scraps are waste. If you allow for the cost of the waste, would you expect the can of least cost to be *x*

*x*

taller or shorter than the one in part (a)? Explain. (c) Estimate the radius, height, and cost of the can in part (b), and determine whether your conjecture was correct. **34.** The designer of a sports facility wants to put a quarter-mile (1320 ft) running track around a football field, oriented as in the accompanying figure on the next page. The football field is 360 ft long (including the end zones) and 160 ft wide. The track consists of two straightaways and two semicircles, with the straightaways extending at least the length of the football field. (a) Show that it is possible to construct a quarter-mile track around the football field. [*Suggestion:* Find the shortest track that can be constructed around the field.] (b) Let *L* be the length of a straightaway (in feet), and let *x* be the distance (in feet) between a sideline of the foot- ball field and a straightaway. Make a graph of *L* ver- sus *x*. (*cont.*) *x*8 in *x*

15 in **Figure Ex-29**

**30.** Repeat Exercise 29 assuming the box is constructed in the

same fashion from a 6-inch-square sheet of metal. **31.** A construction company has adjoined a 1000 ft2 rectan- gular enclosure to its office building. Three sides of the enclosure are fenced in. The side of the building adjacent to the enclosure is 100 ft long and a portion of this side is used as the fourth side of the enclosure. Let *x* and *y* be the dimensions of the enclosure, where *x* is measured parallel to the building, and let *L* be the length of fencing required for those dimensions. (a) Find a formula for *L* in terms of *x* and *y*. (b) Find a formula that expresses *L* as a function of *x* alone. (c) What is the domain of the function in part (b)?

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**0.2 New Functions from Old 15**

(c) Use the graph to estimate the value of *x* that produces the shortest straightaways, and then find this value of *x* exactly. (d) Use the graph to estimate the length of the longest pos-

sible straightaways, and then find that length exactly.

360′

ature *T* and wind speed *v*, the wind chill temperature index is the equivalent temperature that exposed skin would feel with a wind speed of *v* mi*/*h. Based on a more accurate model of cooling due to wind, the new formula is

WCT =

{*T,* 0 ≤ *v* ≤ 3

35*.*74 + 0*.*6215*T* − 35*.*75*v*0*.*16 + 0*.*4275*T v*0*.*16*,* 3 *< v* where *T* is the temperature in ◦F, *v* is the wind speed in mi*/*h, and WCT is the equivalent temperature in ◦F. Find 160′

the WCT to the nearest degree if *T* = 25◦F and (a) *v* = 3 mi*/*h (b) *v* = 15 mi*/*h (c) *v* = 46 mi*/*h.

***Source:*** Adapted from UMAP Module 658, *Windchill*, W. Bosch and L. Cobb, COMAP, Arlington, MA.

**Figure Ex-34**

**38–40** Use the formula for the wind chill temperature index described in Exercise 37. ■ **35–36** (i) Explain why the function *f* has one or more holes in its graph, and state the *x*-values at which those holes occur. (ii) Find a function *g* whose graph is identical to that of *f,* but without the holes. ■ **35.** *f(x)* = *(x* + 2*)(x*2 − 1*)*

*(x* + 2*)(x* − 1*)* **36.** *f(x)* = *x*2 + |*x*|

|*x*|

**38.** Find the air temperature to the nearest degree if the WCT is

reported as −60◦F with a wind speed of 48 mi*/*h. **39.** Find the air temperature to the nearest degree if the WCT is

reported as −10◦F with a wind speed of 48 mi*/*h. **40.** Find the wind speed to the nearest mile per hour if the WCT

is reported as 5◦F with an air temperature of 20◦F. **37.** In 2001 the National Weather Service introduced a new wind chill temperature (WCT) index. For a given outside temper-

✔**QUICK CHECK ANSWERS 0.1**

**1.** (a) [−1*,* + *)* (b) 6 (c) |*t*| + 4 (d) 8 (e) [4*,* + *)* **2.** (a) M (b) I **3.** (a) [−3*,* 3*)* (b) [−2*,* 2] (c) −1 (d) 1 (e) (c) − 34; *w* = √− *A/*2

32 **4.** (a) yes; domain: {65*,* 70*,*71*,* 73*,* 75}; range: {48*,*50*,* 52*,* 56} (b) no **5.** (a) *l* = 2*w* (b) *A* = *l*2*/*2

**0.2 NEW FUNCTIONS FROM OLD**

*Just as numbers can be added, subtracted, multiplied, and divided to produce other numbers, so functions can be added, subtracted, multiplied, and divided to produce other functions. In this section we will discuss these operations and some others that have no analogs in ordinary arithmetic.*

**ARITHMETIC OPERATIONS ON FUNCTIONS** Two functions, *f* and *g*, can be added, subtracted, multiplied, and divided in a natural way to form new functions *f* + *g*, *f* − *g*, *fg*, and *f /g*. For example, *f* + *g* is defined by the formula *(f* + *g)(x)* = *f(x)* + *g(x)* (1)

which states that for each input the value of *f* + *g* is obtained by adding the values of *f* and *g*. Equation (1) provides a formula for *f* + *g* but does not say anything about the domain of *f* + *g*. However, for the right side of this equation to be defined, *x* must lie in the domains of both *f* and *g*, so we define the domain of *f* + *g* to be the intersection of these two domains. More generally, we make the following definition.

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**0.2.1 definition** Given functions *f* and *g*, we define

*(f* + *g)(x)* = *f(x)* + *g(x) (f* − *g)(x)* = *f(x)* − *g(x) (fg)(x)* = *f(x)g(x)*

If *f* is a constant function, that is,

*(f /g)(x)* = *f(x)/g(x)*

*f(x)* = *c* for all *x*, then the product of

For the functions *f* + *g*, *f* − *g*, and *fg* we define the domain to be the intersection *f* and *g* is *cg*, so multiplying a func-

of the domains of *f* and *g*, and for the function *f /g* we define the domain to be the tion by a constant is a special case of multiplying two functions.

intersection of the domains of *f* and *g* but with the points where *g(x)* = 0 excluded (to avoid division by zero).

**Example 1** Let

*f(x)* = 1 + √*x* − 2 and *g(x)* = *x* − 3 Find the domains and formulas for the functions *f* + *g*, *f* − *g*, *fg*, *f /g*, and 7*f* .

***Solution.*** *(f (f (f* + − *(fg)(x) (*7*f /g)(x)* First, *g)(x) g)(x) )(x)* we = = = = = *f(x) f(x) f(x)g(x) f(x)/g(x)* 7*f(x)* will find + − = *g(x) g(x)* 7 the = + = *(*1 formulas = = 71 √+ *(*1 *(*1 + *x x* √+ + √− − *x x* √√2 3 − and − *x x* 2 2

− − *)(x* then 2 2 *) )* − + − the 3*) (x (x* domains. − − 3*)* 3*)* = = *x* 4 The − − *x* 2 formulas + + √√*x x* − − are

2 2 (2) (3)

(4)

(5)

(6)

The domains of *f* and *g* are [2*,*+ *)* and *(*− *,*+ *)*, respectively (their natural domains). Thus, it follows from Definition 0.2.1 that the domains of *f* + *g*, *f* − *g*, and *fg* are the intersection of these two domains, namely,

[2*,*+ *)* ∩ *(*− *,*+ *)* = [2*,*+ *)* (7) Moreover, since *g(x)* = 0 if *x* = 3, the domain of *f /g* is (7) with *x* = 3 removed, namely, [2*,*3*)* ∪ *(*3*,*+ *)*

Finally, the domain of 7*f* is the same as the domain of *f* .

We saw in the last example that the domains of the functions *f* + *g*, *f* − *g*, *fg*, and *f /g* were the natural domains resulting from the formulas obtained for these functions. The following example shows that this will not always be the case.

**Example 2** Show that if *f(x)* = √*x*, *g(x)* = √*x*, and *h(x)* = *x*, then the domain of *fg* is not the same as the natural domain of *h*.

***Solution.*** The natural domain *(fg)(x)* of *h(x)* = √= *xx* √is *x (*− = *x ,*+ = *h(x)*

*)*. Note that

on the domain of *fg*. The domains of both *f* and *g* are [0*,*+ *)*, so the domain of *fg* is

[0*,*+ *)* ∩ [0*,*+ *)* = [0*,*+ *)*

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**0.2 New Functions from Old 17**

by Definition 0.2.1. Since the domains of *fg* and *h* are different, it would be misleading to write *(fg)(x)* = *x* without including the restriction that this formula holds only for *x* ≥ 0.

**COMPOSITION OF FUNCTIONS** We now consider an operation on functions, called *composition*, which has no direct analog in ordinary arithmetic. Informally stated, the operation of composition is performed by substituting some function for the independent variable of another function. For example, suppose that *f(x)* = *x*2 and *g(x)* = *x* + 1

If we substitute *g(x)* for *x* in the formula for *f* , we obtain a new function

*f(g(x))* = *(g(x))*2 = *(x* + 1*)*2

which we denote by *f* ◦*g*. Thus,

*(f* ◦*g)(x)* = *f(g(x))* = *(g(x))*2 = *(x* + 1*)*2

In general, we make the following definition.

Although the domain of *f* ◦*g* may seem complicated at first glance, it makes sense intuitively: To compute *f(g(x))* one needs *x* in the domain of *g* to compute *g(x)*, and one needs *g(x)* in the domain of *f* to compute *f(g(x))*.

**0.2.2 definition** Given functions *f* and *g*, the ***composition*** of *f* with *g*, denoted by *f* ◦*g*, is the function defined by*(f* ◦*g)(x)* = *f(g(x))*

The domain of *f* ◦*g* is defined to consist of all *x* in the domain of *g* for which *g(x)* is in the domain of *f*.

**Example 3** Let *f(x)* = *x*2 + 3 and *g(x)* = √*x*. Find

(a) *(f* ◦*g)(x)* (b) *(g*◦*f )(x)*

***Solution* (*a*)*.*** The formula for *f(g(x))* is

*f(g(x))* = [*g(x)*]2 + 3 = *(*√*x )*2 + 3 = *x* + 3

Since the domain consists of all *x* in of [0*,*+ *g* is [0*,*+ *)* such *)* that and *g(x)* the domain = √*x* of *f* is *(*− *,*+ *)*, the domain of *f* ◦*g* lies in *(*− *,*+ *)*; thus, the domain of *f* ◦*g* is [0*,*+ *)*. Therefore,

*(f* ◦*g)(x)* = *x* + 3*, x* ≥ 0

***Solution* (*b*)*.*** The formula for *g(f(x))* is

*g(f(x))* = √*f(x)* = √*x*2 + 3

Since the domain of *f* is *(*− *,*+ *)* and the domain of *g* is [0*,*+ *)*, the domain of *g*◦*f* consists of all *x* in *(*− *,*+ *)* such that *f(x)* = *x*2 + 3 lies in [0*,*+ *)*. Thus, the domain

Note that the functions *f* ◦*g* and *g*◦*f* in Example 3 are not the same. Thus, the order in which functions are com-

of *g*◦*f* is *(*− *,*+ *)*. Therefore,

*(g*◦*f )(x)* = √*x*2 + 3

posed can (and usually will) make a dif- ference in the end result. There of √*x*2 is + no 3.

need to indicate that the domain is *(*− *,*+ *)*, since this is the natural domain

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Compositions can also be defined for three or more functions; for example, *(f* ◦*g*◦*h)(x)* is computed as *(f* ◦*g*◦*h)(x)* = *f(g(h(x)))*

In other words, first find *h(x)*, then find *g(h(x))*, and then find *f(g(h(x)))*.

**Example 4** Find *(f* ◦*g*◦*h)(x)* if

*f(x)* = √*x, g(x)* = 1*/x, h(x)* = *x*3

***Solution.***

*(f* ◦*g*◦*h)(x)* = *f(g(h(x)))* = *f(g(x*3*))* = *f(*1*/x*3*)* = √1*/x*3 = 1*/x*3*/*2

**EXPRESSING A FUNCTION AS A COMPOSITION** Many problems in mathematics are solved by “decomposing” functions into compositions of simpler functions. For example, consider the function *h* given by

*h(x)* = *(x* + 1*)*2

To evaluate *h(x)* for a given value of *x*, we would first compute *x* + 1 and then square the result. These two operations are performed by the functions

*g(x)* = *x* + 1 and *f(x)* = *x*2

We can express *h* in terms of *f* and *g* by writing

*h(x)* = *(x* + 1*)*2 = [*g(x)*]2 = *f(g(x))*

so we have succeeded in expressing *h* as the composition *h* = *f* ◦*g*.

The thought process in this example suggests a general procedure for decomposing a function *h* into a composition *h* = *f* ◦*g*:

• Think about how you would evaluate *h(x)* for a specific value of *x*, trying to break the evaluation into two steps performed in succession.

• The first operation in the evaluation will determine a function *g* and the second a function *f*.

• The formula for *h* can then be written as *h(x)* = *f(g(x))*.

For descriptive purposes, we will refer to *g* as the “inside function” and *f* as the “outside function” in the expression *f(g(x))*. The inside function performs the first operation and the outside function performs the second.

**Example 5** Express sin*(x*3*)* as a composition of two functions.

***Solution.*** To evaluate sin*(x*3*)*, we would first compute *x*3 and then take the sine, so *g(x)* = *x*3 is the inside function and *f(x)* = sin*x* the outside function. Therefore,

sin*(x*3*)* = *f(g(x)) g(x)* = *x*3 and *f(x)* = sin *x*

Table 0.2.1 gives some more examples of decomposing functions into compositions.

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**0.2 New Functions from Old 19**

**Table 0.2.1** COMPOSING FUNCTIONS

*g*(*x*)

*f*(*x*) FUNCTION INSIDE

OUTSIDE

COMPOSITION

(*x*2 + 1)10 sin3*x* tan (*x*5) √4 − 3*x* 8 + √*x*

*x*2 + 1 sin *x x*54 − 3*x* √*xx* + 1

*x*10 *x*3

tan *x* √*x*8 + *x*

*x* + 1

(*x*2 + 1)10 = *f*(*g*(*x*)) sin3*x* = *f*(*g*(*x*)) tan (*x*5) = *f*(*g*(*x*)) √4 − 3*x* = *f*(*g*(*x*)) 8 + √*x* = *f*(*g*(*x*)) 1

1 *xx* + 1

1

= *f*(*g*(*x*))

**REMARK** There is always more than one way to express a function as a composition. For example, here are two

ways to express *(x*2 + 1*)*10 as a composition that differ from that in Table 0.2.1:

*(x*2 + 1*)*10 = [*(x*2 + 1*)*2]5 = *f(g(x)) g(x)* = *(x*2 + 1*)*2 and *f(x)* = *x*5

*(x*2 + 1*)*10 = [*(x*2 + 1*)*3]10*/*3 = *f(g(x)) g(x)* = *(x*2 + 1*)*3 and *f(x)* = *x*10*/*3

**NEW FUNCTIONS FROM OLD** Car Sales in Millions

The remainder of this section will be devoted to considering the geometric effect of perform- 40

ing basic operations on functions. This will enable us to use known graphs of functions to 3632**Total**

visualize or sketch graphs of related functions. For example, Figure 0.2.1 shows the graphs of yearly new car sales *N(t)* and used car sales *U(t)* over a certain time period. Those 28graphs can be used to construct the graph of the total car sales 24*T (t)* = *N(t)* + *U(t)* 20**Used**

by adding the values of *N(t)* and *U(t)* for each value of *t*. In general, the graph of 16**New**

*y* = *f(x)* + *g(x)* can be constructed from the graphs of *y* = *f(x)* and *y* = *g(x)* by adding 12corresponding *y*-values for each *x*.

841995 2000 2005

***Source:*** NADA.

New

**Figure 0.2.1**

Used

**Example 6** sketch that shows Referring the general to shape Figure of 0.1.4 the graph for the of graphs *y* = √of *x y* = √*x* and *y* = + 1*/x* for *x* ≥ 0.

1*/x*, make a

***Solution.*** To add the corresponding *y*-values of *y* = √*x* and *y* = 1*/x* graphically, just

Use the technique in Example 6 to sketch the graph of the function

√*x* − *x*

1imagine them to be “stacked” on top of one another. This yields the sketch in Figure 0.2.2.

**Figure 0.2.2** Add the *y*-coordinates of obtain the *y*-coordinate of √√*x x* and 1*/x* to

+ 1*/x*.

*y*

*y*

*y*

√*x* + 1/*x*

√*x x*

1/*x*

*x*

*x*

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**TRANSLATIONS** Table 0.2.2 illustrates the geometric effect on the graph of*y* = *f(x)* of adding or subtracting a *positive* constant *c* to *f* or to its independent variable *x*. For example, the first result in the table illustrates that adding a positive constant *c* to a function *f* adds *c* to each *y*-coordinate of its graph, thereby shifting the graph of *f* up by *c* units. Similarly, subtracting *c* from *f* shifts the graph down by *c* units. On the other hand, if a positive constant *c* is added to *x*, then the value of *y* = *f (x* + *c)* at *x* − *c* is *f(x)*; and since the point *x* − *c* is *c* units to the left of *x* on the *x*-axis, the graph of *y* = *f (x* + *c)* must be the graph of *y* = *f(x)* shifted left by *c* units. Similarly, subtracting *c* from *x* shifts the graph of *y* = *f(x)* right by *c* units.

**Table 0.2.2** TRANSLATION PRINCIPLES

**operation on**

Add a positive ***y*** = ***f*(*x*)**

constant *c* to *f*(*x*)

**new equation**

*y* = *f*(*x*) + *c*

**example**

*x*

Before proceeding to the next examples, it will be helpful to review the graphs in Fig- ures 0.1.4 and 0.1.9.

**Example 7** Sketch the graph of

(a) *y* = √*x* − 3 (b) *y* = √*x* + 3 ***Solution.*** *y y* = = √√*x x* − + 3 3 Subtract a positive

Add a positive

Subtract a positive constant *c* from *f*(*x*)

constant *c* to *x*

constant *c* from *x*

*y* = *f*(*x*) − *c*

*y* = *f*(*x* + *c*)

*y* = *f*(*x* − *c*)

**geometric**

Translates the graph of

Translates the graph of

Translates the graph of

Translates the graph of **effect**

*y* = *f*(*x*) up *c* units

*y* = *f*(*x*) down *c* units

*y* = *f*(*x*) left *c* units

*y* = *f*(*x*) right *c* units

*y*

*y* = *x*2 + 2

*y y*

*y*

*y* = *x*2 *y* = (*x* + 2)2 *y* = *x*2 *y* = *x*2

*y* = (*x* − 2)2 2

*y* = *x*2 *y* = *x*2 − 2

*x*

*x*

*x* −2

−2 Using the translation can be obtained by can be obtained by principles given in Table translating the graph of translating the graph of *y y* = = 0.2.2, √√*x x* 2 *y* 3

*x* 9

*y* = √*x*

3

*y*the graph of the equation right 3 units. The graph of

3 12

left 3 units (Figure 0.2.3).

*y* = √*x* − 3

−3 6

*x*

*y*

**Example 8** Sketch the graph of *y* = *x*2 − 4*x* + 5.

3*x*

***Solution.*** Completing the square on the first two terms yields

*y* = *(x*2 − 4*x* + 4*)* − 4 + 5 = *(x* − 2*)*2 + 1

*y* = √*x* + 3

(see Web Appendix H for a review of this technique). In this form we see that the graph can be obtained by translating the graph of *y* = *x*2 right 2 units because of the *x* − 2, and **Figure 0.2.3**

up 1 unit because of the +1 (Figure 0.2.4).

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**0.2 New Functions from Old 21**

**Figure 0.2.4**

*y*

*y* = *x*2

*y*

*y* 8

8 8*x*

*x*

1 *x*

−5 5

−5 2

5 −5 2 5 *y* = (*x* − 2)2 *y* = (*x* − 2)2 + 1

**REFLECTIONS** The graph of *y* = *f(*−*x)* is the reflection of the graph of *y* = *f(x)* about the *y*-axis because the point *(x,y)* on the graph of *f(x)* is replaced by *(*−*x,y)*. Similarly, the graph of *y* = −*f(x)* is the reflection of the graph of *y* = *f(x)* about the *x*-axis because the point *(x,y)* on the graph of *f(x)* is replaced by *(x,*−*y)* [the equation *y* = −*f(x)* is equivalent to −*y* = *f(x)*]. This is summarized in Table 0.2.3.

**Table 0.2.3** REFLECTION PRINCIPLES

3

−6 6

−3*y* = −√*x*

**Example 9** Sketch the graph of *y* = √32 − *x*.

***Solution.*** Using the translation and reflection principles in Tables 0.2.2 and 0.2.3, we can graph obtain of *y* = the √3graph *x* about by the a reflection *y*-axis right 2 units to obtain the graph of followed to obtain the the equation by graph *y* = a translation √of*y* 3−*(x* = − √32*)* −*x*, as = follows: then √32 − translate *x* First (Figure reflect this 0.2.5). graph the

**Figure 0.2.5**

**operation on *y*** = ***f*(*x*)**

**new equation**

**example**

Replace *x* by −*x*

Multiply *f*(*x*) by −1

*y* = *f*(−*x*)

*y* = −*f*(*x*)

**geometric effect**

Reflects the graph of *y* = *f*(*x*) about the *y*-axis

Reflects the graph of *y* = *f*(*x*) about the *x*-axis

*y* = √−*x* 3

*y y* = √*x*

*y*

*x* −6 6

−3*y* = √*xx*

*y*

*y*

*y* 6

6

6

*x*

*x* −10 10

−10 10

−10 10

−6−6−6*y* = √*x* 3

*y* = √−*x* 3 *y* = √2 3 − *x x*

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**Example 10** Sketch the graph of *y* = 4 − |*x* − 2|.

***Solution.*** The graph can be obtained by a reflection and two translations: First translate the graph of *y* = |*x*| right 2 units to obtain the graph of *y* = |*x* − 2|; then reflect this graph about the *x*-axis to obtain the graph of *y* = −|*x* − 2|; and then translate this graph up 4 units to obtain the graph of the equation *y* = −|*x* − 2| + 4 = 4 − |*x* − 2| (Figure 0.2.6).

*y*

*y*

*y*

*y* 88

8

8

*x*

*x*

*x*

*x* −8 8

−6 10

−6 10

−6 10

−8−8−8−8*y* = |*x*|

*y* = |*x* − 2|

*y* = −|*x* − 2|

*y* = 4 − |*x* − 2|

**Figure 0.2.6**

**STRETCHES AND COMPRESSIONS** Multiplying *f(x)* by a *positive* constant *c* has the geometric effect of stretching the graph of *y* = *f(x)* in the *y*-direction by a factor of *c* if *c >* 1 and compressing it in the *y*-

Describe the geometric effect of mul- tiplying a function *f* by a *negative* constant in terms of reflection and

direction by a factor of 1*/c* if 0 *<c<* 1. For example, multiplying *f(x)* by 2 doubles each *y*-coordinate, cuts each *y*-coordinate thereby stretching in half, thereby the graph compressing vertically by the a graph factor vertically of 2, and multiplying by a factor of by 2. 12 stretching or compressing. What is the Similarly, multiplying *x* by a *positive* constant *c* has the geometric effect of compressing geometric effect of multiplying the in- dependent variable of a function *f* by a *negative* constant?

the graph of *y* = *f(x)* by a factor of *c* in the *x*-direction if *c >* 1 and stretching it by a factor of 1*/c* if 0 *<c<* 1. [If this seems backwards to you, then think of it this way: The value of 2*x* changes twice as fast as *x*, so a point moving along the *x*-axis from the origin will only have to move half as far for *y* = *f(*2*x)* to have the same value as *y* = *f(x)*, thereby creating a horizontal compression of the graph.] All of this is summarized in Table 0.2.4.

**Table 0.2.4**

Multiply *f*(*x*) by *c* (*c* > 1)

Stretches the graph of *y* = *f*(*x*) vertically by a factor of *c*

STRETCHING AND COMPRESSING PRINCIPLES

**operation on *y*** = ***f*(*x*)**

**new equation**

**example**

Multiply *f*(*x*) by *c*

Multiply *x* by *c*

Multiply *x* by *c* (0 < *c* < 1)

(*c* > 1)

(0 < *c* < 1)

*y* = *cf*(*x*)

*y* = *cf*(*x*)

*y* = *f*(*cx*)

*y* = *f*(*cx*)

**geometric effect**

Compresses the graph of *y* = *f*(*x*) horizontally by a factor of *c*

Stretches the graph of *y* = *f*(*x*) horizontally by a factor of 1/*c*

2 1*x*

Compresses the graph of *y* = *f*(*x*) vertically by a factor of 1/*c*

*y*

*y*

*y* = cos *x*

*x*

1

*y*

1

*y y* = 2 cos *x y* = cos *x* 1 *y* = 12

cos *x y* = cos *x y* = cos 2*x*

*x y* = cos 12 *xx*

*y* = cos *x*

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**0.2 New Functions from Old 23**

**SYMMETRY** Figure 0.2.7 illustrates three types of symmetries: ***symmetry about the x-axis***, ***symmetry about the y-axis***, and ***symmetry about the origin***. As illustrated in the figure, a curve is symmetric about the *x*-axis if for each point *(x,y)* on the graph the point *(x,*−*y)* is also on the graph, and it is symmetric about the *y*-axis if for each point *(x,y)* on the graph the point *(*−*x,y)* is also on the graph. A curve is symmetric about the origin if for each point *(x,y)* on the graph, the point *(*−*x,*−*y)* is also on the graph. (Equivalently, a graph is symmetric about the origin if rotating the graph 180◦ about the origin leaves it unchanged.) This suggests the following symmetry tests.

**Figure 0.2.7**

*y*(*x*, *y*)

(*x*, –*y*)

Symmetric about the *x*-axis

*y*

(–*x*, *y*) (*x*, *y*)

Symmetric about the *y*-axis

*y*

(–*x*, –*y*)

(*x*, *y*)

*x*

*x*

*x* Explain why the graph of a nonzero function cannot be symmetric about the *x*-axis.

Symmetric about the origin

**0.2.3 theorem (*Symmetry Tests*)**

(*a*) *A plane curve is symmetric about the y-axis if and only if replacing x by* −*x in its*

*equation produces an equivalent equation.*

(*b*) *A plane curve is symmetric about the x-axis if and only if replacing y by* −*y in its*

*equation produces an equivalent equation.*

(*c*) *A plane curve is symmetric about the origin if and only if replacing both x by* −*x*

*and y by* −*y in its equation produces an equivalent equation.*

*y x* = *y*2

**Example 11** Use Theorem 0.2.3 to identify symmetries in the graph of *x* = *y*2.

***Solution.*** Replacing*y* by−*y* yields *x* = *(*−*y)*2, which simplifies to the original equation

*x*

*x* = *y*2. Thus, the graph is symmetric about the *x*-axis. The graph is not symmetric about the*y*-axis because replacing*x* by−*x* yields −*x* = *y*2, which is not equivalent to the original equation*x* = *y*2. Similarly, the graph is not symmetric about the origin because replacing *x* by −*x* and *y* by −*y* yields −*x* = *(*−*y)*2, which simplifies to −*x* = *y*2, and this is again not equivalent to the original equation. These results are consistent with the graph of *x* = *y*2 **Figure 0.2.8**

shown in Figure 0.2.8.

**EVEN AND ODD FUNCTIONS** A function *f* is said to be an ***even function*** if

*f(*−*x)* = *f(x)* (8)

and is said to be an ***odd function*** if

*f(*−*x)* = −*f(x)* (9)

Geometrically, the graphs of even functions are symmetric about the *y*-axis because replac- ing *x* by −*x* in the equation *y* = *f(x)* yields *y* = *f(*−*x)*, which is equivalent to the original

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equation*y* = *f(x)*by (8) (see Figure 0.2.9). Similarly, it follows from (9) that graphs of odd functions are symmetric about the origin (see Figure 0.2.10). Some examples of even func- tions are *x*2*,x*4*,x*6, and cos*x*; and some examples of odd functions are *x*3*,x*5*,x*7, and sin*x*.

*y*

*f*(−*x*) *f*(*x*)

*x* −*x x*

**Figure 0.2.9** This is the graph of an even function since *f(*−*x)* = *f(x)*.

*y*

*f*(−*x*)

*f*(*x*) −*x*

*x*

**Figure 0.2.10** This is the graph of an odd function since *f(*−*x)* = −*f(x)*.

✔**QUICK CHECK EXERCISES 0.2** (*See page 27 for answers.*)

**1.** Let *f(x)* = 3√*x* − 2 and *g(x)* = |*x*|. In each part, give the formula for the function and state the corresponding domain. (a) *f* + *g*: Domain: (b) *f* − *g*: Domain: (c) *fg*: Domain: **2.** (d) *f /g*: Let *f(x)* = 2 − *x*2 and Domain:

*g(x)* = √*x*. In each part, give the formula for the composition and state the corresponding domain. (a) *f* ◦*g*: Domain: (b) *g*◦*f*: Domain:

*x*

**3.** The graph of *y* = 1 + *(x* − 2*)*2 may be obtained by shift-

ing the graph of *y* = *x*2 (left*/*right) by unit(s) and then shifting this new graph (up*/*down) by unit(s). **4.** Let

*f(x)* =

{|*x* + 1|*,* −2 ≤ *x* ≤ 0 |*x* − 1|*,* 0 *< x* ≤ 2

(a) The letter of the alphabet that most resembles the graph

of *f* is . (b) Is *f* an even function?

**EXERCISE SET 0.2** Graphing Utility

**FOCUS ON CONCEPTS**

**1.** The graph of a function *f* is shown in the accompanying figure. Sketch the graphs of the following equations. (a) *y* = *f(x)* − 1 (c) *y* = 12*f(x)* (b) (d) *y y* = = *f(x f* (−− 12*x*1*)* )

*x*

**3.** The graph of a function *f* is shown in the accompanying figure. Sketch the graphs of the following equations. (a) *y* = *f(x* + 1*)* (b) *y* = *f(*2*x)* (c) *y* = |*f(x)*| (d) *y* = 1 − |*f(x)*|

*y*

1

*x*

−1

3 **Figure Ex-3**

**4.** Use the graph in Exercise 3 to sketch the graph of the

equation *y* = *f(*|*x*|*)*. **5–24** ing, *y* Sketch compressing, = 1*/x*, *y* the graph of the equation by translating, = |*x*|, *y* 2

−1

2 **Figure Ex-1**

**2.** Use the graph in Exercise 1 to sketch the graphs of the

following equations. (a) *y* = −*f(*−*x)* (b) *y* = *f(*2 − *x)* (c) *y* = 1 − *f(*2 − *x)* (d) *y* = 12*f(*2*x)*

and stretching or *y* = √3*x* appropriately. the graph of Then *y* = *x*2, *y* reflect- = √*x*, use a graph- ing utility to confirm that your sketch is correct. ■

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**0.2 New Functions from Old 25**

**5.** *y* = −2*(x* + 1*)*2 − 3 **6.** *y* = 12*(x* − 3*)*2 + 2 **7. 9.** *y y* = = *x*3 2 − + √6*x x* + 1 **8. 10.** *y y* = = 1 12*(x*+ 2 √− *x* 2*x* − 4

+ 3*)*

**11.** *y* = 12√*x* + 1 **12.** *y* = −√3*x* **13.** *y* = *x* − 1

3 **14.** *y* = 1 − 1

*x* **15.** *y* = 2 − *x* + 1

1 **16.** *y* = *x* − *x*

1

**17.** *y* = |*x* + 2| − 2 **19. 21. 23.** *y y y* = = = |2*x* 1 2 − + − 2 √3√31| *x x* + + 1 1 **18. 20. 22. 24.** *y y y y* + = = = √1 √√33− *x xx* 2 − − |*x* − 2 2 − 4*x* = − 3| + 3 0

4

**25.** (a) Sketch the graph of *y* = *x* + |*x*| by adding the corre- sponding *y*-coordinates on the graphs of *y* = *x* and *y* = |*x*|. (b) Express the equation *y* = *x* + |*x*| in piecewise form with no absolute values, and confirm that the graph you obtained in part (a) is consistent with this equation. **26.** Sketch the graph of *y* = *x* + *(*1*/x)* by adding correspond- ing *y*-coordinates on the graphs of *y* = *x* and *y* = 1*/x*. Use a graphing utility to confirm that your sketch is correct.

**27–28** Find formulas for *f* + *g,f* − *g,fg*, and *f /g*, and state the **27.** domains *f(x)* = 2of √the *x* − functions. 1, *g(x)* = ■

√*x* − 1 **28. 29.** *f(x)* = Let *f(x)* 1 = + *x*

√*x*2 *x* , and *g(x) g(x)* = 1*x*

= *x*3 + 1. Find (a) *f(g(*2*))* (b) *g(f(*4*))* (c) *f(f(*16*))* **30.** (d) Let (a) *g(x) g(g(*0*)) g(*5*s* = + √2*) x*. Find

(b) (e) *f(*2 *g(*√+ *x* + *h)* 2*)* (f ) *g(*3 + *h).*

(c) 3*g(*5*x)* (d) 1*g(x)* (g) *g(*1*/*√*x )* (e) *g(g(x))* (f ) *(g(x))*2−*g(x*2*)*

(h) *g((x* − 1*)*2*)* (i) *g(x* + *h).*

**31–34** Find formulas for *f* ◦*g* and *g*◦*f*, and state the domains of the compositions. **31. 32.** *f(x) f(x)* = = √*x*2*x* , − *g(x)* 3, ■

= *g(x)* √1 = − √*x*

*x*2 + 3 **33.** *f(x)* = 1 1 + − *x*

*x* , *g(x)* = 1 − *x*

*x* **34.** *f(x)* = 1 + *x*

*x*2 , *g(x)* = *x* 1**35–36** Find a formula for *f* ◦*g*◦*h*. ■ **35.** *f(x)* = *x*2 + 1*, g(x)* = *x* 1*, h(x)* = *x*3 **36.** *f(x)* = 1 + 1

*x , g(x)* = √3*x, h(x)* = 1*x*3

**37–42** Express *f* as a composition of two functions; that is, find *g* and *h* such that *f* = *g*◦*h*. [*Note:* Each exercise has more than **37.** (a) one *f(x)* solution.] = √*x* ■ + 2 **38.** (a) *f(x)* = *x*2 + 1 (b) (b) *f(x) f(x)* = = |*xx* 2 1

− − 3

3*x* + 5|

**39.** (a) *f(x)* = sin2 *x* (b) *f(x)* = 5 + 3

cos *x* **40.** (a) *f(x)* = 3 sin*(x*2*)* (b) *f(x)* = 3 sin2 *x* + 4 sin *x* **41.** (a) *f(x)* = (1 + sin*(x*2*)*)3 (b) *f(x)* =

√1 − √3*x* **42.** (a) *f(x)* = 1

1 − *x*2 (b) *f(x)* = |5 + 2*x*|

**43–46 True–False** Determine whether the statement is true or false. Explain your answer. ■ **43.** The domain of *f* + *g* is the intersection of the domains of

*f* and *g*. **44.** The domain of *f* ◦*g* consists of all values of *x* in the domain

of *g* for which *g(x)* = 0. **45.** The graph of an even function is symmetric about the*y*-axis. **46.** The graph of *y* = *f (x* + 2*)* + 3 is obtained by translating

the graph of *y* = *f(x)* right 2 units and up 3 units.

**FOCUS ON CONCEPTS**

**47.** Use the data in the accompanying table to make a plot

of *y* = *f(g(x))*.

*x* −3−2−10123*f*(*x*) −4−3−2−1012*g*(*x*)

−1

0

1

2

3

−2

−3

**Table Ex-47 48.** Find the domain of *g*◦*f* for the functions *f* and *g* in

Exercise 47. **49.** Sketch the graph of *y* = *f(g(x))* for the functions

graphed in the accompanying figure.

*y*3

*x*

−3 3

*g* −3*f*

**Figure Ex-49**

**50.** Sketch the graph of *y* = *g(f(x))* for the functions

graphed in Exercise 49. **51.** Use the graphs of *f* and *g* in Exercise 49 to esti- mate the solutions of the equations *f(g(x))* = 0 and *g(f(x))* = 0. **52.** Use the table given in Exercise 47 to solve the equations

*f(g(x))* = 0 and *g(f(x))* = 0.

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**53–56** Find*f(x* + *h)* − *f(x)*

*h* and *f(w)* − *f(x) w* − *x* Simplify as much as possible. ■ **53.** *f(x)* = 3*x*2 − 5 **54.** *f(x)* = *x*2 + 6*x* **55.** *f(x)* = 1*/x* **56.** *f(x)* = 1*/x*2 **57.** Classify the functions whose values are given in the accom-

panying table as even, odd, or neither.

−3542

**63.** In each part, classify the function as even, odd, or neither.

(a) *f(x)* = *x*2 (b) *f(x)* = *x*3 (c) *f(x)* = |*x*| (d) *f(x)* = *x* + 1 (e) *f(x)* = *x*5 − *x*

1 + *x*2 (f ) *f(x)* = 2 **64.** Suppose that the function *f* has domain all real numbers. Determine whether each function can be classified as even or odd. Explain. *x* −2−10123*f*(*x*)

3231−35(a) *g(x)* = *f(x)* + 2 *f(*−*x)*

(b) *h(x)* = *f(x)* − 2

*f(*−*x)*

**65.** Suppose that the function *f* has domain all real numbers. *g*(*x*)

1−202−1−4Show that *f* can be written as the sum of an even function *h*(*x*)

−5

8

−2

8

−5

2

and an odd function. [*Hint:* See Exercise 64.]

**Table Ex-57**

**66–67** Use Theorem 0.2.3 to determine whether the graph has **58.** Complete the accompanying table so that the graph of

symmetries about the *x*-axis, the *y*-axis, or the origin. ■ *y* = *f(x)* is symmetric about (a) the *y*-axis (b) the origin.

*x* −3−2 −101 23 *f*(*x*)

1

−1

0

−5

**66.** (a) *x* = 5*y*2 + 9 (b) *x*2 − 2*y*2 = 3

(c) *xy* = 5 **67.** (a) *x*4 = 2*y*3 + *y* (c) *y*2 = |*x*| − 5

(b) *y* = *x* 3 + *x*2 **Table Ex-58 59.** The accompanying figure shows a portion of a graph. Com-

plete the graph so that the entire graph is symmetric about (a) the *x*-axis (b) the *y*-axis (c) the origin. **60.** The accompanying figure shows a portion of the graph of a

**68–69** (i) Use a graphing utility to graph the equation in the first quadrant. [*Note:* To do this you will have to solve the equation for *y* in terms of *x*.] (ii) Use symmetry to make a hand-drawn sketch of the entire graph. (iii) Confirm your work by generating the graph of the equation in the remaining three quadrants. ■

function *f* . Complete the graph assuming that

**68.** 9*x*2 + 4*y*2 = 36 **69.** 4*x*2 + 16*y*2 = 16 (a) *f* is an even function (b) *y*

*f* is an odd function.

*y*

**70.** The graph of the equation *x*2*/*3 + *y*2*/*3 = 1*,* which is shown in the accompanying figure, is called a ***four-cusped hypo- cycloid***. (a) Use Theorem 0.2.3 to confirm that this graph is sym- *x*

*x*

metric about the *x*-axis, the *y*-axis, and the origin. (b) Find a function *f* whose graph in the first quadrant coincides with the four-cusped hypocycloid, and use a **Figure Ex-59**

**Figure Ex-60**

graphing utility to confirm your work. **61–62** Classify the functions graphed in the accompanying fig-

(c) Repeat part (b) for the remaining three quadrants. ures as even, odd, or neither. ■

*y* **61.**

*y*

*y*

*x*

*x*

*x*

(*a*) (*b*)

**Figure Ex-61 62.**

*y*

*y*

Four-cusped hypocycloid **Figure Ex-70**

**71.** The equation *y* = |*f(x)*| can be written as

*y* =

(*a*)

(*b*)

**Figure Ex-62**

*x*

*x*

{ *f(x), f(x)* ≥ 0 −*f(x), f(x) <* 0 which shows that the graph of *y* = |*f(x)*| can be obtained from the graph of *y* = *f(x)* by retaining the portion that lies